Acquisition, (Mis)use and Dissemination of Information: The Blessing of Cursedness and Transparency*

Franz Ostrizek[†] Elia Sartori[‡]

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Abstract

We study strategic interactions when players observe equilibrium statistics, focusing on: First, their endogenous precision as signals of the fundamental; and second, agents' well-documented difficulty in learning from such signals. We define the novel notion of *cursed expectations equilibrium with information acquisition* which disciplines information acquisition in a setting with incorrect learning by means of a *subjective envelope condition*: agents correctly anticipate their actions but incorrectly deem them optimal. Cursed agents use and acquire more private information, which counteracts suboptimal information dissemination and increases welfare. Transparency crowds out private information but is always beneficial; other policy instruments have paradoxical effects.

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†University of Bonn and briq; franz.ostrizek@gmail.com

‡CSEF; elia.sartori@unina.it

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1 Introduction

Many economic decisions are taken in environments with interdependent payoffs and uncertainty both about fundamental states and the actions of others. To guide such decisions, players rely on information that includes direct sources about the fundamental—that may be public or private, exogenously given or acquired at cost—as well as signals about aggregate statistics of the actions of others. Such statistics ("aggregative signals") arise naturally in many economic settings, ranging from the transaction price in a financial market (Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981; Kyle, 1985) and the level of activity and number of infections during a pandemic, to inflation (Lucas, 1972; Morris and Shin, 2005).

Aggregative signals do not just contain information about the actions of others but also disseminate information about the fundamental. The amount of fundamental information conveyed by such statistics depends both on their precision around the true aggregate moment—for which we adopt the moniker of transparency—and on the informativeness of the moment itself, which in turn depends on how much private information the players use and disseminate through their strategies and, ultimately, on how much information they possess to begin with.[1] The intricate interactions resulting from these feedback loops are at the center of lively debates both in policy circles and in the academic literature, especially surrounding policy instruments that directly target the very availability and transmission of information.[2]

One key though hitherto neglected aspect of these interactions is that extracting fundamental information from an equilibrium statistic requires understanding how the actions of others reflect their private information. Ample evidence (e.g. on the winner's curse and underinference in social learning) indicates that agents often fail to take this inferential step.[3] This strongly suggests that a complete analysis of such environment should take into account both the endogenous dissemination of information as well agents' limited grasp of the information contained in aggregative signals.

Towards this end we study a beauty contest game with information acquisition and adapt cursed equilibrium (Eyster and Rabin, 2005) as a tractable model of incomplete inference from aggregative signals. Cursed equilibrium provides a parsimonious solution concept capturing the range from rational to fully cursed agents who fail to take into account that the actions of other players are a result of their private information and hence consider the aggregate outcomes to be uninformative about fundamentals. It has been used successfully to account for overbidding in common value auctions and has also found applications to financial markets (Eyster et al., 2019). We consider the simple specification in which agents target a combination of the state and the average action to focus on the

[1] Deviations from full transparency may arise, e.g., from measurement error, intentional coarsening of information or delays in reporting. For example, a fully transparent financial market would be one where a trader knows the transaction price before submitting his order. A lower level of transparency would correspond to a market where traders have only noisy information about the transaction price, say they observe the current price at another similar market place or in the past.

[2] We briefly review some of these debates in the related literature.

[3] In the winner's curse, agents fail to appreciate that they are more likely to win a common value auction when their private information leads them to overestimate the value of the prize. Implicitly, they dismiss the information contained in the fact that they won the auction. See Kagel and Levin (2002) for a review of the experimental evidence. For social learning, see Weizsäcker (2010).

novel interaction between cursedness, transparency, and information acquisition. In our analysis, cursedness and transparency serve as complementary antagonists: Transparency determines how much aggregative information is available (and therefore how much can be ignored) while cursedness represents the degree of ignorance. Without transparency there is no aggregative information and hence nothing to be cursed about, while at the other extreme fully cursed agents completely ignore aggregative information and hence transparency is without effect.

Agents in our model also make an information acquisition decision: before playing the beauty contest, they choose (at a cost) the precision of their private information about the state. To make this decision, clearly, they have to assess the value of information. As cursed agents fail to understand the information environment, one needs to take a stand on how such agents perceive this value, i.e. if and to what extent they are aware of their future misuse. Devising such a notion while attaining the two goals of behavioral plausibility and analytical tractability presents considerable challenges. To the best of our knowledge, information acquisition by cursed agents has not been considered in the literature. We propose a notion of *cursed expectations equilibrium with information acquisition* that obeys the following behavioral assumption: Agents correctly anticipate their expected welfare and play, but they are not meta-rational, i.e. they do not consider their future information use to be erroneous. The latter implies that agents follow a subjective envelope condition which allows for a highly tractable analysis of the information acquisition problem.

We now preview our results. The equilibrium is characterized by a vector of loadings on the different sources of information. As agents become more cursed, they substitute away from the aggregative signal and increase the use of private information. This is because cursedness makes them perceive the aggregative signal as less informative so they need to rely more on their remaining information sources. The subjective envelope condition implies that the use of private information is a sufficient statistic for its acquisition; in particular, the comparative statics of information acquisition and information use coincide. Moreover, the acquisition channel creates a new feedback loop as the precision of private information becomes a function of its perceived value: using and acquiring more information, cursed agents disseminate it more effectively.

Cursedness and transparency have opposed impacts on the equilibrium loadings: For any degree of cursedness, an increase in transparency makes agents substitute from private information towards the aggregate signal but crowds out private information acquisition. The crowding out effect, however, never overturns the direct positive effect and the aggregative signal becomes more informative about the state as transparency increases. This monotonicity does not necessarily hold for other measures of informational efficiency, such as the total precision of information available to agents or the realized covariance between the aggregate action and the state. Along the latter metric, cursedness increases the inflow of private information into the aggregative signal, but it hinders its extraction and thereby reduces the efficiency of dissemination. With information acquisition, these forces balance exactly and – contrary to the intuition that inferential naivete hampers information aggregation and to the result with exogenous private information both in this paper and Eyster et al. (2019) – the covariance between the aggregate action and the state is independent of cursedness and transparency. While cursed agents lean against the wind coming from aggregative information, they inject more private information.

The tractability of our framework allows an exhaustive analysis of the welfare consequences of cursedness and of information policies. In the rational equilibrium, the use and acquisition of private information is inefficiently low because of an information dissemination externality. If agents are cursed enough, however, they may use (and acquire) at or even above the efficient level, though they simultaneously misuse their signals. Indeed, welfare is nonmonotonic in the degree of cursedness. Local to rationality, cursedness is bliss: An increase in cursedness causes a welfare gain from improved dissemination that dominates the welfare loss from privately suboptimal use. The former is a first-order gain as dissemination is inefficiently low, while the latter is second order, as the information use of rational agents is privately optimal. Welfare in the fully cursed case, however, is always lower than in the rational benchmark, as such agents ignore the aggregative signal completely and therefore do not reap any gains from information dissemination.

Lower information acquisition costs and more public information have an ambiguous effect. Both increase welfare in the rational benchmark for our payoff specification (Bayona, 2018; Colombo et al., 2014), but can reduce welfare with partially cursed agents: They cause agents to substitute away from the aggregative signal and towards private information or the fundamental source (resp.), exacerbating suboptimal information use when agents are partially cursed. We show that this effect can dominate the direct effect of cheaper and more precise information, causing the paradoxical comparative statics. More transparency, by contrast, always increases welfare. It unambiguously improves the dissemination of information and does not exacerbate cursed agents' misuse.

Although their welfare increases with transparency, cursed agents fail to reap its full benefits. How does an agent who is able to extract all the information from the environment interact with a cursed crowd? Could it be beneficial to act in an environment with less rational agents? We address these questions by studying the behavior of an atomistic rational agent—such as proverbial smart money in a financial market—facing equilibrium play in a economy of cursed agents. Such a shrewd agent benefits from the large amount of information disseminated by the cursed crowd, sometimes even abstaining from acquiring private information. However, the shrewd agent is also harmed by their misuse of information as strategic complementarity forces him to follow the crowd and distort his actions away from the fundamental. Compared to the rational environment, low levels of cursedness are always beneficial for the shrewd agent (whose welfare can even exceed first best). At high levels of cursedness the imitation effect dominates in games with sufficiently strong strategic complements, making excessive cursedness harmful. The trade-off between information free riding and miscoordination creates nontrivial comparative statics in the policy parameters. A shrewd agent always profits from transparency, but can be hurt by more public information and lower information acquisition costs, even when they are beneficial for the cursed crowd.

We conclude the introductory section by discussing the related literature. In Section 2 we present the model. We establish existence and uniqueness of a cursed equilibrium in Section 3, taking the precision of private information as given, and briefly discuss comparative statics results. We introduce information acquisition in Section 4, defining the notion of cursed expectations equilibrium with information acquisition. Section 5 analyses the positive comparative statics of the model. We turn to welfare analysis in Section 6. We analyze the optimal strategy and welfare of an atomistic rational agent in Section 7. Section 8 concludes.

1.1 Related Literature

Conducting our analysis within the workhorse class of linear quadratic models, we connect to a rich theoretical and applied literature. Models within this class (and those that exhibit a similar best response structure) are used to investigate questions of information in a wide range of applications, ranging from in business cycles (e.g. Hellwig and Veldkamp, 2009; Angeletos and La'O, 2010; Benhabib et al., 2015), demand function competition (Vives, 1988, 2017), to political economy (e.g. Shadmehr et al., 2018).

The study of the social value of information in this setting has been initiated by Morris and Shin (2002), who show that more precise public information can reduce welfare in games with strategic complementarities as it leads to excessive coordination. Angeletos and Pavan (2007) characterize the inefficiencies of information use in a general linear-quadratic Gaussian game. Ui and Yoshizawa (2015) classify such games according to the welfare properties of additional public and private information. Colombo et al. (2014) study how private information acquisition affects the value of information, establishing a close link between efficient acquisition and efficient use. All these papers consider exogenous information, i.e. signals of the fundamental. We analyze the value of information in the presence of a signal of the average action providing information of endogenous precision about the state. Bayona (2018) considers an information structure with such a signal in a setting akin to Angeletos and Pavan (2007), establishing that this can lead to a dissemination inefficiency in the use of private information.[4] We analyze the interplay with information acquisition – indeed, our rational benchmark nests a payoff restriction of Colombo et al. (2014), Bayona (2018), and their insofar unexplored meet – as well as agents' limited understanding of aggregative information. Our restriction of the payoff structure to the simple beauty contest game allows us to isolate the novel sources of inefficiency in our setting.

The results of Morris and Shin (2002) have spurred extensive debate in the literature about the desirability of public information in particular in the context of central bank announcements.[5] This discussion has often been couched in the terminology of "anti-transparency" vs. "pro-transparency". This label does not correspond to our usage, as we reserve the word transparency for the precision of the public signal about the aggregate

[4] Amador and Weill (2012) show that with such a dissemination externality more public information can cause a decrease in welfare, even without interdependent payoffs.

[5] Svensson (2006), e.g., argues that the ratio of private to public precision required for the paradoxical welfare result is unreasonably high and Woodford (2005) calls into question the assumptions on strategic complementarity and welfare. The role of these assumptions is clarified and general conditions for such welfare results are given in Angeletos and Pavan (2007) and Ui and Yoshizawa (2015). Cornand and Heinemann (2008) instead analyze the extensive margin in public information provision and show that more precise public information is always desirable if it reaches the optimal fraction of agents. Kool et al. (2011) show that public information can reduce information acquisition by market participants and thereby increase financial market volatility.

action.[6] Although we would argue that much of the information provided by central banks is aggregative in nature and explore the impact of such transparency at length, we also contribute to the original debate by demonstrating a novel channel based on cursed inference which can render public *fundamental* information undesirable: It distracts behavioral agents from other information sources whose information content they underestimate. The issue of endogenous information dissemination has been studied in the context of business cycles by Wong (2008) who show that increased transparency can be self-defeating as it reduces the information available to the central bank itself to learn about the state of the economy, a mechanism that has also been studied in Morris and Shin (2005).[7]

Inference from a signal that aggregates information contained in individual best responses is also at the center of the literature on information aggregation in financial markets.[8] Grossman and Stiglitz (1980) show that the equilibrium informativeness of the price system is unresponsive to changes in transparency: an increase in noise leads to more information acquisition which exactly offsets the direct effect. We establish a similar invariance along a certain metric of informational efficiency in our setting, but show that transparency has an impact on the total precision available to (rational) agents.

Cursed equilibrium was proposed by Eyster and Rabin (2005) as a model of underinference from the actions of others to explain the winner's curse as well as trade in settings where Bayes-Nash equilibrium would predict a breakdown due to adverse selection. Eyster et al. (2019) apply a cursed analogue to rational expectations equilibrium in a trading game and show that cursed behavior can explain excessive trade volume. We adapt cursed equilibrium to a beauty contest game with endogenous information augmenting it with an information acquisition stage. To the best of our knowledge, the present paper is the first to analyze information acquisition with cursed agents.

2 The Model

The game has two stages: First, agents choose how much private information to acquire. Second, agents play a beauty contest game. We begin our description of the setting with the second stage and treat the game with *exogenous* precision of private information in this and the following section, and add information acquisition in Section 4.

[6] In the financial economics literature, enhanced transparency is sometimes conceptualized as the sharing of private signals between asymmetrically informed traders, e.g. Glosten and Milgrom (1985); Chowdhry and Nanda (1991); Pagano and Volpin (2012). Pagano and Röell (1996) define transparency as the extent to which market makers can observe the size and direction of the current order flow, a notion that is much closer to that we use in this paper. They find that greater transparency generates lower trading costs for uninformed traders on average, although not necessarily for every size of trade.

[7] Amador and Weill (2010) show that through a similar signal jamming channel public information can be welfare decreasing, as it reduces the informativeness of the price system thereby increasing uncertainty about the monetary shock.

[8] In demand function competition, Vives (2017) shows that there is both an information dissemination externality as well as a pecuniary externality, the latter causing excessive weight on private information.

Actions and Payoff

There is a unit interval of agents $i \in [0,1]$, playing a simple beauty contest game. Their payoff is given by

$$u(a_i, \bar{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(a_i - \bar{a})^2, \tag{1}$$

where $a_i \in \mathbb{R}$ is the action of player i, $\bar{a} = \int_i a_i \, \mathrm{d}i$ is the average action[9] and $\theta \in \mathbb{R}$ is the state (or fundamental). We allow for both strategic complementarity ($r > 0$), and substitutability ($r < 0$), and assume that complementarity is not too strong ($r < 1$) to ensure the existence of a unique interior linear equilibrium and a planner solution.

The restriction to a simple beauty contest allows us to isolate the inefficiencies generated by the features specific to our information environment: the dissemination externality of aggregative information, and cursed updating from that source. Indeed, in our simple beauty contest game both information use and acquisition are efficient for the rational benchmark without aggregative information (Angeletos and Pavan, 2007; Colombo et al., 2014).[10]

Signals and Inference

The following information structure is common knowledge. The state θ is drawn from the prior distribution $\mathcal{N}(0, \tau_\theta^{-1})$.[11] Agents receive three signals: a *private fundamental* signal $s_i = \theta + z_{s_i} \sim \mathcal{N}(\theta, \tau_s^{-1})$, i.i.d. across agents, a *public fundamental* signal $y = \theta + z_y \sim \mathcal{N}(\theta, \tau_y^{-1})$ about the state and a *public aggregative* signal $p = \bar{a} + z_p \sim \mathcal{N}(\bar{a}, \tau_p^{-1})$ where τ_p, the precision of the aggregative signal as a signal of $\bar{a}$, is our transparency parameter.[12] We will endogenize τ_s in the information acquisition stage.

The optimal action is given by

$$a_i(s_i, y, p) = \arg\max_{a_i} \mathbb{E}_i \left[u(a_i, \bar{a}, \theta) \right] \tag{2}$$

where $\mathbb{E}_i$ is the expectation operator with respect to agent i's information, including his updating biases.[13] As u is quadratic, (2) takes the linear best response form

$$a_i = (1-r)\mathbb{E}_i(\theta) + r\mathbb{E}_i(\bar{a}) \tag{3}$$

[9] As is customary we adopt a SLLN for the private signals as a convention, see Vives (2008, 10.3.1) for a discussion. One formal operationalization of this is to view the integrals in the sense of Pettis, see Uhlig (1996).

[10] This contrasts with the specification of the beauty contest in Morris and Shin (2002) who consider the utility function $u(a_i, \bar{a}, \theta) = -(1-r)(a_i - \theta)^2 - r(a_i - a_j)^2$, which results in a dependence of individual utility on the variance of others' actions. A natural extension of our paper is to analyze the interplay between a richer payoff structure (which broadens the set of economic applications) and our information environment.

[11] A prior mean of zero is merely a convenient normalization. We insist on a proper prior as we analyze the comparative statics of ex-ante welfare.

[12] The situation we have in mind is that of a central authority having exclusive access to the set of actions chosen by each player inside a market. With those data she can perform statistical analysis (difficult because of missing data, imperfect reports, etc) and produce a report which will then be observed without further noise by everyone. Interpreted as the accuracy of the process turning actions into a report, transparency becomes a natural parameter for positive comparative statics as well as policy evaluation.

[13] While we assume that the aggregative signal is observed before taking the action, our model can be written equivalently in action schedules $a_i : p \mapsto a \in \mathbb{R}$ (see Vives, 2014). Both formulations are equivalent for rational and (partially) cursed agents and it depends on the application which seems more natural. In the context of (financial) markets, it is common to analyze models of demand/supply function competition, while acting based on a realized signal seems more natural for individual consumers or workers reacting to the inflation rate.

Throughout, we focus on linear equilibria. That is, the optimal action rule takes the form

$$a_i = \alpha_0 + \alpha_1 s_i + \alpha_2 y + \alpha_3 p \tag{4}$$

for some vector of loadings $\boldsymbol{\alpha}$. Then, we can write the true aggregate action as

$$\bar{a} = \int_0^1 a_i \, \mathrm{d}i = \delta_0 + \delta_1 \theta + \delta_2 y + \delta_3 p \tag{5}$$

with aggregate weights $\boldsymbol{\delta}$. Inspection of equation (5) makes clear that the aggregative signal p provides information of *endogenous precision* about θ. Indeed, under the assumption that $\delta_3 \neq 1$ (and conditionally on y), p is informationally equivalent to

$$\hat{p} = \frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3} y\right] - \frac{\delta_0}{\delta_1} = \theta + \frac{1}{\delta_1} z_p \sim \mathcal{N}\left(\theta, \frac{1}{\delta_1^2 \tau_p}\right) \tag{6}$$

The Bayesian posterior on θ can be written based on the three conditionally independent sources $(s, y, \hat{p})$ which determines the posterior on $\bar{a}$ through (5). The precision of the aggregative signal about the state, $\delta_1^2 \tau_p$, depends both on transparency τ_p and on the equilibrium loading δ_1.

Cursed Equilibrium

As a model of the failure to update from observing the action of others, we adapt cursed equilibrium (Eyster and Rabin, 2005). In this solution concept, agents are characterized by a parameter χ, the degree of cursedness, that ranges from $\chi = 0$ for rational benchmark to $\chi = 1$ denoting fully cursed behavior. A fully cursed agent fails to perceive any correlation between other agents' actions and their private information. Instead, he thinks that others play according to the marginal distribution of their actions conditional on his private information. Consequently, according to the beliefs of a fully cursed agent i with information I_i, the action of agent j is

$$a_j = \mathbb{E}[a_j | \mathrm{I}_i] = \alpha_0 + \alpha_1 \mathbb{E}[\theta | \mathrm{I}_i] + \alpha_2 y + \alpha_3 p + \alpha_1 \left(s_j - \mathbb{E}[\theta | \mathrm{I}_i]\right) \tag{7}$$

where the α_k are the weights used in the linear strategy of player j. The fully cursed agent treats the prediction error $s_j - \mathbb{E}[\theta | \mathrm{I}_i]$ as *independent* of the state. Therefore, in a linear symmetric equilibrium,

$$\bar{a} = \delta_0 + \delta_1 \mathbb{E}[\theta | \mathrm{I}_i] + \delta_2 y + \delta_3 p \tag{8}$$

i.e. that the aggregate statistic is independent of θ conditional on his information. A fortiori, $\bar{a}$ and hence p do not provide additional information about the state.[14]

[14] In contrast to other updating biases – e.g. overconfidence or dismissiveness –, cursed agents correctly perceive the relative precision of p as a signal about the aggregate action. They fail, however, to relate it to the private information of others and to extract information about the state.

Partially cursed agents are characterized by an interior level of cursedness $\chi \in (0,1)$. They form expectations as a convex combination of rational and fully cursed ones, namely

$$\mathbb{E}_\chi[\theta|I_i] = \chi \frac{\tau_y y + \tau_s s_i}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_y y + \tau_s s_i + \delta_1^2 \tau_p \hat{p}}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p} \tag{9}$$

$$\mathbb{E}_\chi[\bar{a}|s_i, y, p] = \chi\left(\delta_0 + \delta_1 \frac{\tau_y y + \tau_s s_i}{\tau_\theta + \tau_y + \tau_s} + \delta_2 y + \delta_3 p\right) + (1-\chi)(\delta_0 + \delta_1 \theta + \delta_2 y + \delta_3 p) \tag{10}$$

Note that even cursed agents do all the updating about the state θ and then turns it into a belief about $\bar{a}$ through the equilibrium condition (5): they have "equilibrium awareness". Where they go wrong is in the under-appreciation of the correlation between their private information and others' actions.

When we interpret our game as one of submitting action schedules as a function of the aggregative signal (see FN 13), cursedness also captures agents inability to engage in conditional or hypothetical thinking (Esponda and Vespa, 2014; Ngangoué and Weizsäcker, 2021). In light of this evidence, we interpret the degree of cursedness not as an individual characteristic but as codetermined by the market structure.

Cursed equilibrium is defined as a solution concept for Bayesian games. Due to our information structure, however, the model described so far is not a Bayesian game, strictly speaking: agents react to a signal that itself is an integral over actions. We therefore adapt cursed equilibrium in a fashion similar to a linear rational expectations equilibrium:[15]

Definition 1. A vector of loadings $(\boldsymbol{\alpha}, \boldsymbol{\delta})$ constitutes a χ-*cursed expectations equilibrium* if $\boldsymbol{\alpha}$ satisfies the best response condition (3)-(4) with expectations formed according to (9)-(10) given $\boldsymbol{\delta}$; and the aggregate action is consistent with individual actions, $\boldsymbol{\delta} = \boldsymbol{\alpha}$.

3 Equilibrium Analysis for Exogenous τ_s

This section studies the equilibrium without information acquisition. An equilibrium is computed by matching coefficients in the best-response function (3).

Proposition 1. *There exists a unique χ-cursed equilibrium for any τ_s. It is given by*

$$\alpha_0 = \delta_0 = 0 \tag{11}$$

$$\alpha_1 = \delta_1 \tag{12}$$

$$\alpha_2 = \delta_2 = \frac{\delta_1^2 \tau_y}{(1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)} \tag{13}$$

$$\alpha_3 = \delta_3 = 1 - \frac{\delta_1 (1-r)\tau_s}{(1-r)\tau_s - \delta_1\left(\tau_\theta + \tau_y\right)} \tag{14}$$

where $\delta_1 \in [0,1)$ is the unique real solution to

$$\delta_1 = [1 - r + r\delta_1]\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p}\left(1 + \chi \frac{\delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s}\right) \tag{15}$$

[15] See also Eyster et al. (2019) for a similar approach in a trading game with finitely many agents.

In (15) the RHS has a natural interpretation as it denotes the optimal loading on private information given aggregate δ_1. First, the private signal is valuable for predicting the state, with best-response weight $1-r$, as well as the aggregate action to the degree that it reflects the state (conditional on public signals), with best-response weight $r\delta_1$. Second, the relative precision of the private signal is the usual Bayesian weight $\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}$. Hence, the private signal is ignored ($\delta_1 = 0$) only if it is pure noise ($\tau_s = 0$). This term also contains the information spillover effect: the more other agents use their private information (higher δ_1), the more can be learned from the aggregative signal, which reduces the weight on the private signal. Third, the final term adjusts this weight as cursed agents fail to understand that the aggregative signal is informative about the state and therefore perceives the private signal to be *relatively* more informative. In the extreme case of $\chi = 1$, the agent ignores the aggregative signal and the two final factors simplify to $\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}$, the relative precision of the private signal as if there were no aggregative information. Therefore, transparency is without effect in the fully cursed equilibrium while, symmetrically, cursedness has an impact on the equilibrium[16] only if there is an informative aggregative signal ($\tau_p > 0$). Cursedness manifests itself as a pure updating bias and only distorts inference from the aggregative signal. Absent such signal, cursed agents act just like a rational agent as they correctly interpret all fundamental sources of information.

At first sight, that cursedness matters only in the presence of an aggregative signal may be surprising when compared with the implications of cursed equilibrium in a common value auction. In the auction, there is no aggregative information available to the agent before he chooses his action but still cursedness impacts his choice. This, however, is a natural consequence of the payoff structure: In an auction, the agent considers his payoff conditional on winning the auction, which is exactly such an aggregative conditioning event. In our model, the payoffs themselves weigh all states equally ex-ante and there is no such "implicit conditioning" embedded in them.

The fully rational case is easily obtained from Proposition 1 but doesn't lead to a simple and immediately interpretable representation. It was analyzed in depth in Bayona (2018). The opposite case results in a considerable simplification.

Corollary (**Fully Cursed Equilibrium**). *The equilibrium with* $\chi = 1$ *is*

$$\delta_1^{FC} = \frac{(1-r)\tau_s}{\tau_\theta + \tau_y + (1-r)\tau_s}, \quad \delta_2^{FC} = \frac{\tau_y}{\tau_\theta + \tau_y + (1-r)\tau_s}, \quad \delta_3^{FC} = 0 \tag{16}$$

The role of strategic substitutability and complementarity is directly apparent in the fully cursed equilibrium. If there are no such strategic interactions, cursed agents weigh the two signals at their (mental) disposal according to their precision. Strategic complementarity shifts weight away from the private signal s_i and towards the public signal, y, while substitutability has the opposite effect.

The fact that the fully cursed equilibrium puts no weight on the aggregative signal deserves a clarification. This does not follow from cursedness alone. Indeed even for fully cursed agents, the aggregative signal, p, remains a valid source of the public fundamental

[16] Although both cursedness and transparency affect all three loadings, they enter δ_2 and δ_3 only indirectly as summarized by δ_1 in this convenient representation of the equilibrium system.

signal, y, and of public noise, z_p. As those are relevant for coordination purposes, agents want to incorporate p into their best response as long as others do so, but always with a lower weight. Hence, only as a result of the interplay between equilibrium and cursedness, do we obtain $\delta_3^{FC} = 0$ by an unraveling argument.[17]

3.1 Comparative Statics

We study the impact of cursedness and transparency on the equilibrium loadings as well as on $\delta_1^2 \tau_p$, the precision of the aggregative signal p as a signal about θ.

Proposition 2. *The comparative statics are given by*

$$\frac{\partial \delta_1}{\partial \chi} \geq 0, \qquad \frac{\partial \delta_2}{\partial \chi} \geq 0, \qquad \frac{\partial \delta_3}{\partial \chi} \leq 0, \qquad \frac{\partial}{\partial \chi} \frac{\delta_1}{\delta_2} \leq 0 \tag{17}$$

$$\frac{\partial \delta_1}{\partial \tau_p} \leq 0, \qquad \frac{\partial \delta_2}{\partial \tau_p} \leq 0, \qquad \frac{\partial \delta_3}{\partial \tau_p} \geq 0, \qquad \frac{\partial}{\partial \tau_p} \frac{\delta_1}{\delta_2} \geq 0, \tag{18}$$

all inequalities being strict if $\tau_p \neq 0$ *and* $\chi \neq 1$. *Furthermore,*

$$\frac{\partial}{\partial \tau_p} \delta_1^2 \tau_p > 0. \tag{19}$$

Cursed agents rely less on the aggregative signal as their behavioral bias makes them underappreciate its information content and substitute towards both private and public information. It shifts relative loadings on fundamental information in favor of the public signal ($\frac{\partial}{\partial \chi} \frac{\delta_1}{\delta_2} \leq 0$) irrespective of other parameters, in particular of the degree of complementarities r. This is because the public signal is a closer substitute to the aggregative one as both have a public noise component.

The mirror structure of comparative statics in Proposition 2 confirms the intuition that cursedness and transparency are complementary antagonists: increasing the processing bias has qualitatively the same impact on equilibrium loadings as reducing the amount of information provided by this source. In particular, higher transparency decreases the loading on private information: As the private information of others is disseminated more effectively, I rely less on my own. Nevertheless, this crowding out effect never dominates and the precision of the aggregative signal about the fundamental is always increasing in transparency.

Proposition 3. *The weight on private information respond to parameter changes as follows:*

$$\frac{\partial \delta_1}{\partial \tau_s} \geq 0, \qquad \frac{\partial \delta_1}{\partial \tau_y} \leq 0, \qquad \frac{\partial \delta_1}{\partial r} \leq 0 \tag{20}$$

Agents rely more on the private signal as it becomes more precise relative to the public fundamental signal. As complementarities become stronger the public signals become more

[17] In a related paper, Vives (2017) considers limited inference, equivalent to fully cursed behavior, in a LQN model of competition in supply schedules with unknown costs. Fully cursed traders in his setting do not ignore the noisy signal of fundamentals, the price, as it is directly payoff relevant.

attractive relative to the private signal as it allows for better coordination with the aggregate action. Consequently, an increase in r decreases the weight on the private signal.

At the heart of the analysis is the loading on private information δ_1, as it determines the endogenous precision of the aggregative signal. The other two loadings, δ_2 and δ_3 have an immediate interpretation as the weight given by the agent to the public fundamental and aggregative signal, respectively, but have to be interpreted with care. The agent loads on the public signal, for example, both directly through the public signal as well as indirectly through the aggregative signal. Hence, the comparative statics of those loadings in τ_s, τ_y, r (which are all ambiguous, see Appendix) are difficult to interpret. To get a better understanding of how agents use public information, we study equilibrium loadings in the fundamental representation

$$a_i = \underbrace{\frac{\delta_1 + \delta_2}{1 - \delta_3}}_{\beta}\theta + \delta_1 z_{s_i} + \underbrace{\frac{\delta_2}{1 - \delta_3}}_{\gamma_2} z_y + \underbrace{\frac{\delta_3}{1 - \delta_3}}_{\gamma_3} z_p \tag{21}$$

The regression coefficient of the individual action (and hence also the aggregate action) on the state is denoted β. This parameter hence can be interpreted as a measure of informational efficiency of the equilibrium. The weights γ_2, γ_3 on the public shocks differ from the direct loadings on the signals by a factor of $\frac{1}{1-\delta_3}$, since the aggregative signal contains and amplifies both public shocks.[18] Using (13)-(15) one obtains

Proposition 4. *In equilibrium, the loadings in the fundamental representation (21) are*

$$\beta = 1 - \frac{\delta_1 \tau_\theta}{(1-r)\tau_s}, \qquad \gamma_2 = \frac{\delta_1 \tau_y}{(1-r)\tau_s}, \qquad \gamma_3 = \frac{1-\delta_1}{\delta_1} - \frac{\left(\tau_\theta + \tau_y\right)}{(1-r)\tau_s}.$$

Furthermore,

$$\frac{d\beta}{d\chi} < 0, \qquad \frac{d\beta}{d\tau_p} > 0, \qquad \frac{d\beta}{d\tau_s} > 0, \qquad \frac{d\gamma_2}{d\tau_y} > 0, \qquad \frac{d\gamma_2}{d\tau_s} < 0, \qquad \frac{d\gamma_2}{d\tau_p} < 0.$$

The responsiveness of the action to the state, β, is determined both by the use of private information and by the efficiency of its dissemination. If the precision of private information or the level of transparency increases, it rises unambiguously. An increase in cursedness increases private information use but hampers dissemination as cursed agents fail to learn from the aggregative signal. The latter effect dominates and the responsiveness decreases unambiguously. The comparative statics of γ_2 are intuitive: As the public fundamental signal becomes relatively more precise, agents substitute towards it.[19] The comparative statics of γ_3 are identical to those of δ_3 and are therefore ambiguous. For instance, consider the effect of an increase in τ_s. If τ_s is low, the information content of the aggregative signal is

[18] The average action does not contain the agents private noise term z_s, whence the direct loading and the weight on the private shock coincide and we do not introduce a new variable γ_1.

[19] The comparative static of δ_2, by contrast, can be ambiguous. Take for example the impact of τ_s, where we can have $\frac{\partial \delta_2}{\partial \tau_s} > 0$: As δ_1 increases and agents substitute away from aggregative information, they desire to keep a similar loading on public information. Before, this was obtained as a byproduct of aggregative information, but now has to be used directly through δ_2. The transformation to γ_2 neutralizes this composition effect.

low as well and increases strongly together with τ_s. Therefore, agents substitute towards the aggregative signal and γ_3 increases. If τ_s is large, agents already possess precise information about the state and the information content of the aggregative signal reacts relatively little to an increase in τ_s. Therefore, agents substitute away from the noisy aggregative signal and γ_3 goes down.

4 Information Acquisition

In the first stage, agents simultaneously choose the precision of their private signal, τ_s, at cost $c\tau_s$. The crucial step to study this decision is deriving a representation for the agents perceived ex-ante welfare as a function of τ_s. This is tricky because cursed agents fail to understand the information environment. We propose a notion of *cursed expectations equilibrium with information acquisition* based on three principles. First, cursedness is a bias of conditional thinking and inference, individuals are correct on average and unconditionally. Therefore, at the information acquisition stage, agents conceptualize their true ex-ante welfare as a function of parameters, their actions and their equilibrium conjecture. Second, cursedness is the result of a systematic tendency and not of a systematic but unexpected mistake: agents correctly anticipate their information use, but they do not consider it to be erroneous. Therefore, agents believe that the cursed use of information is (individually) optimal and so evaluate private information following a subjective envelope condition. In other words, agents do not try and fix their bias via information acquisition: they consider only the direct impact of more information holding their actions fixed. Third, when evaluating the returns to private acquisition an agent holds coplayers' acquisition and use fixed at their equilibrium values. All in all, in a cursed expectations equilibrium with information acquisition agents consider the gradient of true ex-ante welfare taking their future actions and the equilibrium relation as given.

We now proceed towards a formal definition. The true ex-ante welfare of an agent that acquires precision τ_s, plays according to $\boldsymbol{\alpha}$ and faces an equilibrium $\boldsymbol{\delta}$ is given by

$$W(\boldsymbol{\alpha},\boldsymbol{\delta},\tau_s) = \mathbb{E}\left[-(1-r)(a_i-\theta)^2 - r(a_i-\bar{a})^2\right] - c\tau_s \tag{22}$$

$$= -\frac{\alpha_1^2}{\tau_s} - (1-r)\left(\left[\alpha_2 + \delta_2\alpha_3\frac{1}{1-\delta_3}\right]^2\frac{1}{\tau_y} + \left[\alpha_3\frac{1}{1-\delta_3}\right]^2\frac{1}{\tau_p} + \left(\alpha_1+\alpha_2+\alpha_3\frac{1}{1-\delta_3}\{\delta_1+\delta_2\}-1\right)^2\frac{1}{\tau_\theta}\right) - c\tau_s \tag{23}$$

The optimal τ_s taking both the equilibrium loadings as given as well as considering the continuation play as optimal solves

$$\frac{\partial}{\partial\tau_s}W(\boldsymbol{\alpha},\boldsymbol{\delta},\tau_s) = 0. \tag{24}$$

Hence, we arrive at the first-order *subjective envelope condition*

$$\frac{\alpha_1^2}{\tau_s^2} = c \tag{25}$$

The weight on private information in the best response, α_1, is a sufficient statistic for the marginal value of private information, even if agents are cursed. Cursedness only affects the calculus through $\alpha_1(\delta_1)$ and equilibrium. This envelope condition follows from the rational choice of a Bayesian agent, both in our setting and in the case without an aggregative signal but a more general payoff structure studied in Colombo et al. (2014). For the cursed case, we include the condition as part of our equilibrium notion.

Definition 2. A tuple $(\boldsymbol{\alpha}, \boldsymbol{\delta}, \tau_s)$ constitutes a χ-*cursed expectations equilibrium with information acquisition* if $(\boldsymbol{\alpha}, \boldsymbol{\delta})$ constitute a χ-cursed expectations equilibrium given τ_s and (α_1, τ_s) satisfy the subjective envelope condition (25).

The subjective envelope condition (25) and equilibrium consistency give a linear dependence between τ_s and δ_1

$$\tau_s = \frac{\delta_1}{\sqrt{c}} \tag{26}$$

Taking account of endogenous information acquisition in the equilibrium condition (15), we arrive at

$$\delta_1 = [1 - r + r\delta_1] \frac{\delta_1}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)} \left(1 + \chi \frac{\sqrt{c}\delta_1^2 \tau_p}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}\right) \tag{27}$$

Contrary to the game with exogenous τ_s, the existence of an interior equilibrium requires a condition on parameters. This is because we need to ensure that agents are willing to acquire private information, i.e. that the best-response weight on private information (RHS) exceeds δ_1 local to $\delta_1 = 0$. This is the case if

$$\sqrt{c} \leq \frac{1 - r}{\tau_\theta + \tau_y} \tag{28}$$

or, equivalently, if the costs of acquiring information are sufficiently small compared to the benefits of private information in the trivial candidate equilibrium. These benefits depend on the precision of prior and public information $\tau_\theta + \tau_y$ and the relative value of public versus private information, as summarized by $1 - r$.[20] If this condition is not met, we are stuck in a corner solution with zero information acquisition (and therefore use). Note that (27) has a trivial solution, which describes the equilibrium in that case. We now summarize this discussion.

[20] If we instead assume convex costs with an Inada-type condition at zero, the analogue of (28) is always satisfied and we have an interior solution.

Proposition 5. *There exists a unique χ-cursed equilibrium with information acquisition. If $\sqrt{c} < \frac{(1-r)}{\tau_y+\tau_\theta}$, it is given by*

$$\alpha_0 = \delta_0 = 0 \tag{29}$$

$$\alpha_1 = \delta_1 \tag{30}$$

$$\alpha_2 = \delta_2 = \frac{\sqrt{c}\delta_1\tau_y}{(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)} \tag{31}$$

$$\alpha_3 = \delta_3 = 1 - \frac{\delta_1(1-r)}{(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)} \tag{32}$$

$$\tau_s = \frac{\delta_1}{\sqrt{c}} \tag{33}$$

and $\delta_1 \in (0,1)$ is the unique interior real solution to (27). Otherwise, we have a corner equilibrium with $\delta_1 = \delta_3 = \tau_s = 0$ and $\delta_2 = \frac{\tau_y}{\tau_\theta+\tau_y}$.

Discussion: Cursed Information Acquisition

Before moving to the analysis of cursed equilibrium with information acquisition, we discuss in more detail the behavioral assumptions that lead to our notion and contrast them with possible alternatives.

Recall that the information acquisition choice requires agents to take an ex-ante perspective and consider the value of information before the signals have realized. Cursed equilibrium, however, is defined directly using the conditional expectation ex-interim. To derive the subjective value of private information, we therefore need to take a stance on how cursed agents perceive their information environment and their actions from an ex-ante perspective. In cursed expectations equilibrium with information acquisition, agents take the precision of public information as well as the equilibrium loadings as given and have correct beliefs about their realized equilibrium welfare, however they do not attempt to use information acquisition to fix their bias.

A quasi-Bayesian specification in which agents have a misspecified prior but are Bayesian otherwise shares some of these features. Such an agent uses information identically to a χ-cursed agent for every signal realization and we hence have an envelope theorem if his perceived signal precisions are given by[21]

$$\left(\widehat{\tau}_\theta, \widehat{\tau}_s, \widehat{\tau}_y, \widehat{\tau}_p\right) = \left(\tau_\theta, \tau_s, \tau_y, (1-\chi)\,\tau_p \frac{\tau_\theta+\tau_y+\tau_s}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right). \tag{34}$$

However, imposing such a quasi-Bayesian perspective upon the model has several drawbacks. First, the perceived precision of the exogenous signal about the aggregate action now depends on the endogenous equilibrium weight δ_1. Second, and more importantly, the perceived precision of the public signal p depends on his *individually chosen* τ_s. In other words, the agent behaves as if his personal information acquisition affects the precision

[21]The quasi-Bayesian representation is unique among those preserving the conditional independence of all signals and the (implicit) weight on the prior mean.

of public information obtained by all agents. In addition, this is inconsistent with taking equilibrium as given: Since δ_1 depends on the precision of the aggregate signals (but only *population* τ_s), taking $\widehat{\tau}_p$ to be endogenous but δ_1 to be exogenous to your private decision is problematic.

A second alternative is to derive information acquisition by maximizing true ex-ante welfare without imposing the subjective envelope condition. This corresponds to a rational agent who correctly predicts his cursed actions but desires to correct them by distorting the precision of private information available to his future biased self. This level of sophistication together with an inability to use the aggregative signal correctly ex-interim seems implausible. Our notion instead allows us to avoid this excessive meta-rationality while preserving a subjective world view that is consistent with the atomistic position of the agent within the game.

5 Comparative Statics with Information Acquisition

The information acquisition channel introduces a confounding force to the comparative statics of Section 3. Consider a parameter change that – holding acquisition fixed – would increase δ_1 (for example, higher cursedness). Since the precision of the aggregative signal increases with δ_1, this depresses the value of private information. This feedback puts downward pressure on the acquisition and use of private information. In equilibrium, however, τ_s and δ_1 are tightly linked by the envelope condition (25) and this feedback loop never dominates.

Proposition 6. *It holds that*

$$\frac{\partial \delta_1}{\partial c} < 0.$$

The comparative statics wrt. other parameters in Propositions 2 and 3 continue to hold with endogenous information acquisition.

The comparative statics of τ_s have the same sign as the comparative statics of δ_1, namely

$$\frac{\partial \tau_s}{\partial c} \leq 0, \qquad \frac{\partial \tau_s}{\partial \tau_y} \leq 0, \qquad \frac{\partial \tau_s}{\partial \tau_p} \leq 0, \qquad \frac{\partial \tau_s}{\partial r} \leq 0 \tag{35}$$

The precision of privately acquired information is reduced by both higher cost and more precise alternative sources, for any degree of cursedness. The effect of cursedness and transparency on δ_1 are preserved qualitatively and amplified: An increase in cursedness, for instance, does not only affect δ_1 through information use, but also causes an increase in information acquisition, further increasing δ_1. In particular, the endogenous precision of the aggregative signal about the state, $\delta_1^2 \tau_p$, is still increasing in transparency (see Figure 1). The mirrored roles of transparency and cursedness – providing aggregative information and dampening its processing – continue to be in place and lead to opposed comparative statics in these two parameters.

As discussed in the case without information acquisition, the comparative statics of δ_2 are often ambiguous since it captures only part of the fundamental loading on the public fundamental signal. We hence return to the fundamental representation of a_i derived in (21).

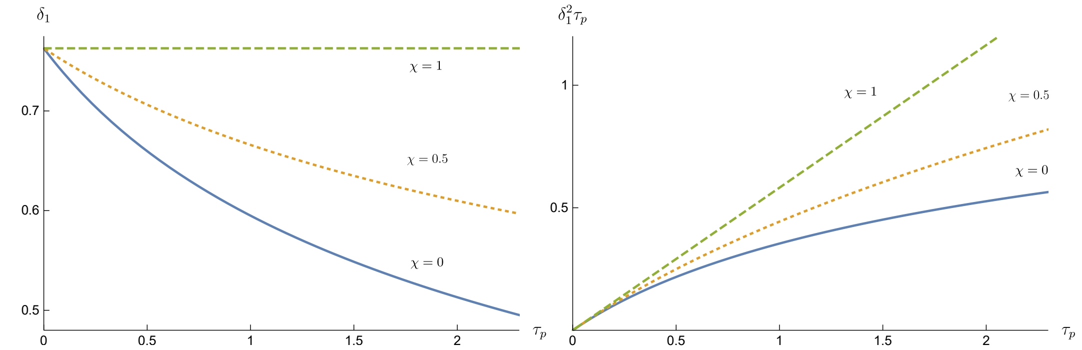

Figure 1: Crowd-out vs Reliance on Private Information: The effect of τ_p on δ_1 and $\delta_1^2\tau_p$ for different levels of cursedness.

Proposition 7. *In an equilibrium with information acquisition, the loadings in the fundamental representation (21) are*

$$\beta = 1 - \frac{\sqrt{c}\tau_\theta}{1-r}, \quad \gamma_2 = \frac{\tau_y\sqrt{c}}{1-r}, \quad \gamma_3 = \frac{1}{\delta_1}\left(1 - \delta_1 - \frac{\tau_\theta + \tau_y}{1-r}\sqrt{c}\right). \tag{36}$$

The comparative statics of β and γ_2 are immediate. γ_3 is decreasing in cursedness and increasing in transparency.

Only cursedness and transparency have an unambiguous impact on $\gamma_3 = \frac{\delta_3}{1-\delta_3}$: when agents consider the aggregative signal to be less informative, either because they are more cursed or because the environment is less transparent, they will use it less. The comparative statics of γ_3 in $\tau_y, \tau_\theta, r, c$ are instead ambiguous. Consider, for example, the effect of an increase in c. On the one hand, it reduces information acquisition and makes agents substitute towards the aggregative signal. On the other hand, the reduction in information acquisition and reliance on the private signal removes the very basis of information in p, making it less attractive. Either effect can dominate. A similar intuition is the basis for the ambiguous comparative statics in the other parameters: the precision of the aggregative signal changes both in level and relative to the precision of other signals.

The state-action regression coefficient β is decreasing in costs, the prior precision and the degree of complementarity; it does not depend on either cursedness or transparency. Recall that when private precision was fixed (Proposition 4), β was increasing in τ_p and decreasing in χ. Once we allow agents to adjust τ_s in response to changes in the perceived value of private information caused by either a more transparent environment or a decrease in cursedness, those effects are neutralized. The resulting invariance property has three consequences of economic relevance: First, we cannot identify the degree of cursedness in a market by just looking at the responsiveness of individual actions to fundamentals. Second, transparency is an ineffective tool at increasing the informational efficiency along the β metric as its effect is fully offset by lower acquisition and use of private information, a result akin to the invariance with respect to the variance of net supply from noise traders in

Grossman and Stiglitz (1980). Third, as opposed to the setting without information acquisition and the findings in Eyster et al. (2019), cursedness does not reduce the responsiveness of the aggregate action with respect to the true state. Even though cursed agents reduce the efficiency of information dissemination by failing to amplify the information content of p, they inject more private information into the system.

Since information acquisition stabilizes the ratio $\frac{\delta_1}{\tau_s}$ to $\sqrt{c}$, γ_2 is now pinned down by the triplet τ_y, r, c with intuitive comparative statics: agents load more on the public fundamental signal if it is more precise, if private acquisition is more costly, or if the coordination motive is stronger. The degree of cursedness does not affect γ_2, not even indirectly. Even though cursed agents fail to process all information disseminated through the aggregative signal, when they can adjust τ_s, their increased demand for and use of private information exactly offsets the less efficient inference.

While cursedness does not affect the total weight on information obtained through private signals, $\beta - \gamma_2$, it changes its composition: agents substitute away from indirect inference of disseminated private information, δ_3, towards information agents have acquired themselves, $1 - \delta_3$. This decomposition is apparent in the following rewriting of (21)

$$a_i = (\beta - \gamma_2)\left[(1 - \delta_3)(\theta + z_s) + \delta_3 \theta\right] + \gamma_2 y + \gamma_3 z_p \tag{37}$$

and is depicted in Figure 2.

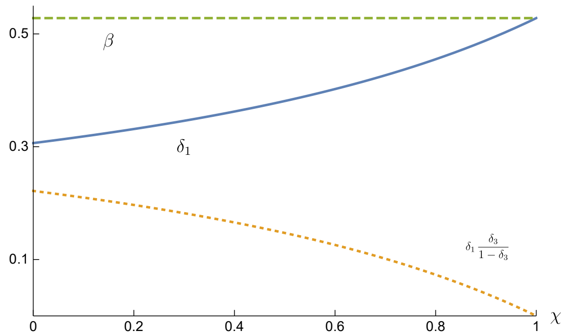

Figure 2: Equilibrium weight on information from private signals: direct and indirect.

Total Precision

We continue to address the impact of transparency on the informational efficiency of the environment. The measures $\delta_1^2 \tau_p$ and β analyzed in the previous section combine information acquisition, dissemination and use and therefore might not be the ideal metric to assess the informativeness of the environment. We hence study the impact of τ_p on $\tau_\Sigma := \tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p$, the total precision of information that agents possess about the state.[22] This impact is ambiguous, at least qualitatively: The endogenous precision of the

[22] A similar question is addressed in Morris and Shin (2005) who show that a central bank may inadvertently sabotage its own information collection from observing aggregate outcomes by providing precise information about these outcomes to the market.

aggregative signal is increasing in transparency while the precision of private information is decreasing.

To quantify the counteracting effects it is convenient to consider the following factorization of total precision (obtained by manipulating (27))

$$\tau_\Sigma = \left(\frac{1 - r + r\delta_1}{\sqrt{c}}\right)\left(1 + \chi\frac{\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s}\right). \tag{38}$$

Recall that δ_1 and τ_s are decreasing in τ_p while $\delta_1^2\tau_p$ is increasing. Thus the first factor in (38) is increasing in transparency if and only if actions are substitutes ($r < 0$), while the second factor is always increasing, strictly unless $\chi = 0$. Therefore transparency increases total precision in the rational benchmark if actions are substitutes and decreases it if actions are complements. If actions are substitutes, transparency increases τ_Σ even for interior degrees of cursedness. In a game of complementarities, cursedness increases the range of parameters where transparency is desirable by scaling up the second factor: Cursedness dampens the crowding out of agents' private information as a response to the increase in transparency. Summarizing,

Proposition 8. *The total precision available the agents is increasing in τ_p if and only if $r < \mathrm{R}(\chi)$, for a cutoff $\mathrm{R}(\chi)$, possibly trivial, with $\mathrm{R}(0) = 0$ and $\mathrm{R}' > 0$.*

Figure 3 shows the effects of both τ_p and τ_y on total precision as a function of the strategic complementarity parameter r. It is easy to see that more precise public fundamental information has a impact qualitatively comparable to transparency but the opposite interaction with cursedness.

Proposition 8 implies that there exists a threshold level of cursedness $\bar{\chi}$ for each fixed r (trivially equal to zero if $r \leq 0$) such that transparency increases total precision if and only if $\chi > \bar{\chi}$. When cursedness is large, aggregative information becomes more effective at enhancing total precision as the crowding out effect is shut down. In particular, in a fully cursed economy τ_Σ is always increasing in τ_p as there is no crowding out of private information acquisition and use. As agents become more cursed, however, this metric is less and less relevant as a welfare measure since they cannot reap all gains from a more informative environment. We hence turn to welfare analysis to evaluate this trade-off.

6 Welfare

We first characterize the first-best benchmark for the use and acquisition of information. Then we identify and characterize the inefficiencies of the χ–cursed equilibrium with information acquisition. Finally, we perform welfare comparative statics.

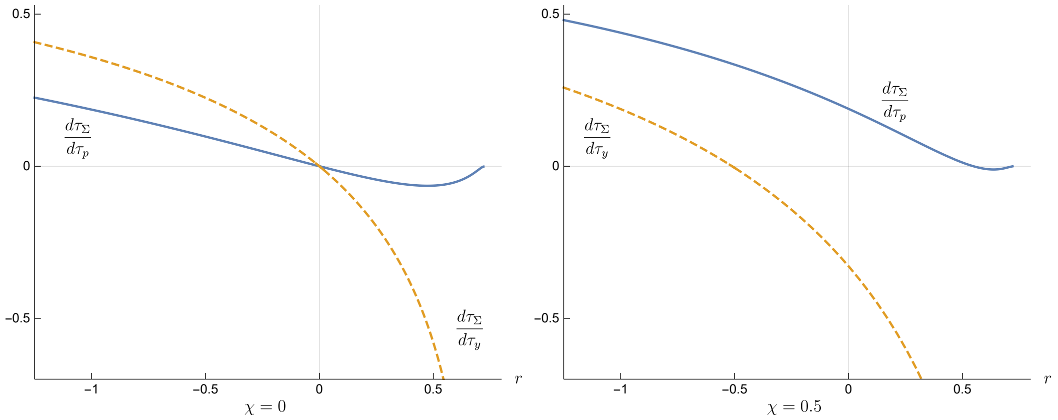

Figure 3: The effect of τ_p, τ_y on τ_Σ in the rational (left) and partially cursed ($\chi = 0.5$, right) model.

6.1 The Planner Problem

As a welfare benchmark, we consider the problem of a planner who controls both the use and acquisition of information, but cannot share information across agents.[23] To this end we impose the consistency condition $\alpha = \delta$ in the welfare expression (22) and, with slight abuse of notation, let $W(\delta, \tau_s) := W(\delta, \delta, \tau_s)$ denote the objective function of a planner choosing

$$\left(\delta^\star, \tau_s^\star\right) = \arg\max_{(\delta, \tau_s)} W(\delta, \tau_s) \tag{39}$$

where straightforward calculations show

$$W(\delta, \tau_s) = -\frac{(1-r)}{(1-\delta_3)^2}\left\{\frac{\delta_2^2}{\tau_y} + \frac{\delta_3^2}{\tau_p} + \frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right\} - \frac{\delta_1^2}{\tau_s} - c\tau_s. \tag{40}$$

We proceed by characterizing the solution of (39).

Proposition 9. *The efficient linear action rule satisfies*

$$\delta_2^\star = \frac{\tau_y(1-\delta_1^\star)}{\tau_\theta + \tau_y + \tau_p\delta_1^\star}, \quad \delta_3^\star = \frac{\delta_1^\star(1-\delta_1^\star)\tau_p}{\tau_\theta + \tau_y + \tau_p\delta_1^\star}, \quad \left(\frac{\delta_1^\star}{\tau_s^\star}\right)^2 = c \tag{41}$$

where $\delta_1^\star$ is the unique solution of

$$\delta_1 = (1-r+r\delta_1)\,\tau_s^\star \frac{1}{\underbrace{\left(\frac{\tau_\theta + \tau_y + \tau_p\delta_1^2}{\tau_\theta + \tau_y + \tau_p\delta_1}\right)}_{\textit{efficiency wedge}}\left(\tau_\theta + \tau_y + \tau_p\delta_1^2\right) + \tau_s^\star} \tag{42}$$

Condition (42) corresponds to the rational equilibrium condition (15) modified by an efficiency wedge, accounting for the fact that using public information as the basis of action

[23] This is the benchmark customarily adopted in the literature (Angeletos and Pavan, 2007). It avoids the unfair comparison with an economy in which agents can also share information: as there are uncountably many, this would coincide with playing a game of complete information with a trivial solution and trivial welfare properties.

dilutes the dissemination of private information. The planner internalizes this effect and therefore downweighs public information by the adjustment term $\frac{\tau_\theta+\tau_y+\tau_p\delta_1^2}{\tau_\theta+\tau_y+\tau_p\delta_1} < 1$. This wedge is equal to one only if $\delta_1^\star = 1$ and therefore $\delta_2^\star = \delta_3^\star = 0$, i.e. if the aggregative signal is not polluted by public signals to begin with, which is impossible in equilibrium. This observation has two economically relevant implications. First, the efficient solution features a higher weight on private information compared to the rational equilibrium.[24] Therefore, the equilibrium with $\chi = 0$ is never efficient. For a positive level of cursedness, the inefficiency of the equilibrium is an immediate consequence of the processing bias. Therefore, we have the second implication, the equilibrium is always inefficient.

The optimality condition for τ_s is our familiar envelope condition: The use of private information is a sufficient statistic for the gains from acquiring it, even for the planner. Both efficient and equilibrium information acquisition are fully determined by the respective use of private information.

Proposition 10. *There is under-(over)acquisition of private information in equilibrium if and only if there is under- (over)use of private information in equilibrium, i.e.*

$$\operatorname{sgn}\{\tau_s - \tau_s^\star\} = \operatorname{sgn}\{\delta_1 - \delta_1^\star\}. \tag{43}$$

In particular, information acquisition in equilibrium is efficient if an only if information use in equilibrium is efficient.

Before turning to other inefficiencies of equilibrium we record the comparative statics of first-best welfare. Plugging the optimality conditions (41) into the welfare expression (40) we get

$$W^\star := W\left(\delta^\star, \tau_s^\star\right) = \max_{\delta_1} -2\sqrt{c}\delta_1 - \frac{(1-r)(1-\delta_1)^2}{\tau_\theta + \tau_y + \delta_1^2\tau_p} \tag{44}$$

The comparative statics of first-best welfare now follow easily from an envelope argument.

Proposition 11. *First-best welfare satisfies*

$$\frac{dW^\star}{d\tau_\theta} > 0, \quad \frac{dW^\star}{d\tau_y} > 0, \quad \frac{dW^\star}{d\tau_p} > 0, \quad \frac{dW^\star}{dc} < 0.$$

As information is used efficiently by the planner, increasing precision – whatever the source – or lowering acquisition costs always increases first best welfare.

6.2 The Inefficiencies of Equilibrium

In equilibrium, by contrast, information is generally used inefficiently: agents do not internalize the dissemination externality and they are subject to a processing bias that makes them misuse available information.

[24] The ratio of δ_2 to δ_3 is the same in the efficient action rule and in the rational equilibrium. While agents in the rational equilibrium underuse and underacquire private information, the relative weights on public aggregative and fundamental information are efficient. This changes for the cursed equilibrium, since $\frac{d}{d\chi}\frac{\delta_2}{\delta_3} > 0$ (Proposition 6). Whenever agents are cursed, they overweigh fundamental information relative to aggregative information.

Proposition 12. *The rational equilibrium always has inefficiently low information acquisition. Sufficiently cursed agents acquire more private information than the efficient benchmark if* $\tau_p > \overline{\tau}_p$, *where*

$$\overline{\tau}_p = \left(\tau_\theta + \tau_y\right) \frac{1 - 2\delta_1^{FC}}{\left(\delta_1^{FC}\right)^3}. \tag{45}$$

Since rational agents do not internalize the dissemination externality, the equilibrium with $\chi = 0$ features underacquisition. As τ_s is increasing in χ by Proposition 3, cursedness alleviates this inefficiency. This effect can be strong enough to lead to overacquisition relative to the efficient benchmark if the aggregative signal is sufficiently precise. Intuitively, if transparency exceeds the lower bound (45), then dissemination is so effective that even $\tau_s^\star$ (which is independent of χ) is low compared to the precision of information acquired by the agent in the fully cursed equilibrium (which is by construction independent of τ_p). Then, there exists an interior χ such that the equilibrium use and acquisition of private information coincide with the efficient quantity (see the left panel of Figure 4).[25] Even in this case, however, agents misperceive the information environment and hence misuse their information. To analyze this source of inefficiency, consider the gradient of welfare at equilibrium as we vary the cursedness parameter (displayed in the right panel of Figure 4).

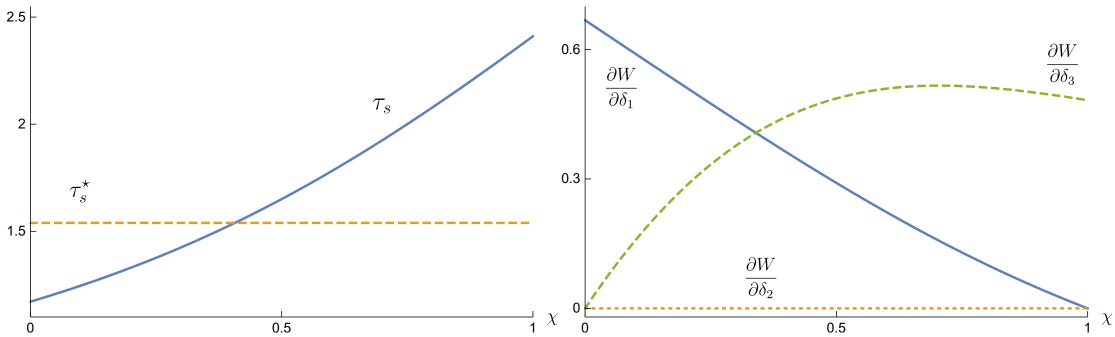

Figure 4: Equilibrium vs. efficient information acquisition (Proposition 12, left) and the gradient of W in equilibrium (Proposition 13, right) as a function of cursedness.

Proposition 13. *In a rational equilibrium,* δ_3 *is conditionally efficient:* $\frac{\partial W}{\partial \delta_3}(\delta, \tau_s) = 0$, *for* $\chi = 0$.

In a fully cursed equilibrium, δ_1 *is conditionally efficient:* $\frac{\partial W}{\partial \delta_1}(\delta, \tau_s) = 0$, *for* $\chi = 1$.

In any equilibrium, δ_2 *is conditionally efficient:* $\frac{\partial W}{\partial \delta_2}(\delta, \tau_s) = 0$, *for all* χ.

In a fully rational equilibrium, the only externality is the dissemination of private information. Fixing the use of private information and thereby its dissemination, the other loadings of the equilibrium are conditionally efficient. In a fully cursed equilibrium agents ignore the aggregative signal altogether, so there is no dissemination externality and private

[25] The cutoff (45) is always met ($\overline{\tau}_p < 0$) if incentives for private information acquisition are sufficiently high, namely $\frac{\sqrt{c}\left(\tau_\theta + \tau_y\right)}{1 - r} \le \frac{1}{2}$.

information is used efficiently.[26] Independently of the degree of cursedness, there is no externality or misunderstanding in the use of the public fundamental signal.

6.3 The Comparative Statics of Equilibrium Welfare

The sources of equilibrium inefficiency identified in Propositions 12 and 13 provide the bedrock for analyzing the impact of cursedness and changes in the information environment on equilibrium welfare. Let $W^{EQ} := W\left(\delta^{\chi}, \tau_s^{\chi}\right)$ denote equilibrium welfare, where we introduce $\delta^{\chi}, \tau_s^{\chi}$ as shorthand for the equilibrium with χ-cursed agents.

Cursedness is Bliss

Consider a marginal increase in cursedness starting from the rational equilibrium. It has two impacts on welfare. First, agents now use their information suboptimally as they underestimate the information contained in p. The associated welfare reduction is second order, however, as $\boldsymbol{\delta}$ is privately optimal in the rational equilibrium. Second, cursed agents acquire and disseminate more information. Since the rational equilibrium features underacquisition, this impact on the dissemination externality has a first order effect on welfare. Thus, local to rationality and up to first order, cursedness only has beneficial effects on welfare. When χ is already large, however, marginal increments in cursedness have a first order negative effects from additional misuse, while the underacquisition gap is narrower if existent at all. The inefficient use dominates close to full cursedness. Fully cursed agents do not use the information contained in the aggregative signal, so providing more disseminated information to them is not valuable.

Proposition 14 (Cursedness is Bliss)**.**

$$\left.\frac{dW^{EQ}}{d\chi}\right|_{\chi=0} > 0 \tag{46}$$

Furthermore,

$$\left.\frac{dW^{EQ}}{d\chi}\right|_{\chi=1} < 0 \tag{47}$$

so any level of cursedness maximizing equilibrium welfare must be interior.

The shape of welfare as a function of χ and the comparison to efficient welfare is shown in Figure 5. One might wonder if the comparison in the plot holds in general or whether a fully cursed economy can ever outperform full rationality. This cannot happen. Indeed, it is easy to show that in the two extreme cases $\chi \in \{0, 1\}$, welfare takes the simple form

$$W^{EQ} = -\sqrt{c}\,(1 + \delta_1). \tag{48}$$

and it follows from the comparative statics of δ_1 that the fully cursed equilibrium has lower welfare than the rational case: Even though acquisition and dissemination of private infor-

[26]Again, recall that fully cursed equilibrium coincides at the action stage with fully rational equilibrium in which τ_p is set to zero. Without an aggregative signal our model is a special case of Angeletos and Pavan (2007) where payoffs satisfy the conditions for efficient use of information.

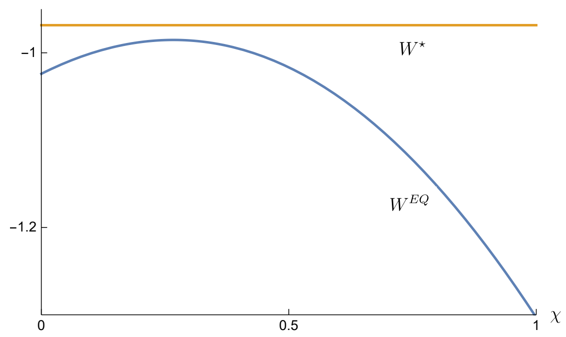

Figure 5: Cursedness is Bliss

mation are higher, cursed agents are unable to make any use of their aggregative information. The inefficiently imprecise aggregative information provided in the rational equilibrium is preferable to complete ignorance of – albeit plentiful – aggregative information.

The Impact of Information on Equilibrium Welfare

In contrast to the efficient solution (Proposition 11), more information and lower costs do not always increase welfare in equilibrium.

Proposition 15. *If χ is sufficiently close to either* 0 *or* 1 *or τ_p is sufficiently small,* W^{EQ} *is increasing in τ_y, τ_θ and decreasing in c. If, however, τ_p is sufficiently large, there exist an interior region of χ such that equilibrium welfare is*

- *decreasing in τ_y, τ_θ if strategic complementarities are sufficiently strong ($r > \frac{1}{2}$),*
- *and increasing in c in a game with strategic substitutes ($r < 0$).*

Equilibrium welfare is always increasing in τ_p.

The proposition identifies sufficient conditions for counterintuitive comparative statics. Let us first focus on the comparative static with respect to the precision of the public fundamental signal.[27] An increase in τ_y has two equilibrium effects. First, a direct effect as the precision of the agents' information increases mechanically. Second, a substitution effect as agents reduce their use and acquisition of private information and also substitute towards y and away from p. When strategic complementarities are sufficiently strong, the second effect is particularly important and causes the paradoxical comparative static: For partially cursed agents, the use of p is already suboptimally low and a further decrease entails a welfare loss. This effect dominates the welfare calculus for interior χ. For (close to) fully cursed agents, however, the substitution effect is negligible as they disregard p and the direct effect is important as they rely heavily on y.[28]

[27] A similar result is also obtained in Morris and Shin (2002), but for different reasons. There, all signals are fundamental, but the increased use of public information entails a payoff externality. In our setting, payoffs are such that – absent dissemination externality and cursedness – information use is efficient (Angeletos and Pavan, 2007; Colombo et al., 2014) and hence more precise public information is always welfare improving. Both ingredients are needed to break this result, it continues to hold even if $\tau_p > 0, \chi = 0$, a case not subsumed by the literature.

[28] We obtain further analytical insight by studying the limit as $\tau_p \to \infty$ (Appendix A). From (62), welfare is decreasing in τ_y (or, τ_θ) if and only if $1 - 2r + r\chi < 0$, which provides a lower bound, $\chi > 2 - \frac{1}{r}$, that can be satisfied

Likewise, an increase in acquisition costs potentially benefits partially cursed agents: It causes them to rely less on private information and to substitute towards p, whose informativeness they effectively underestimate. This effect can dominate with sufficiently strong strategic substitutes, when the value of information is relatively low because agents want to anti-coordinate. As we can see in Figure 6, the effect is present for sufficiently small costs, since then the substitution is towards a relatively informative p, whereas if costs are too high, the aggregative signal itself becomes too noisy.

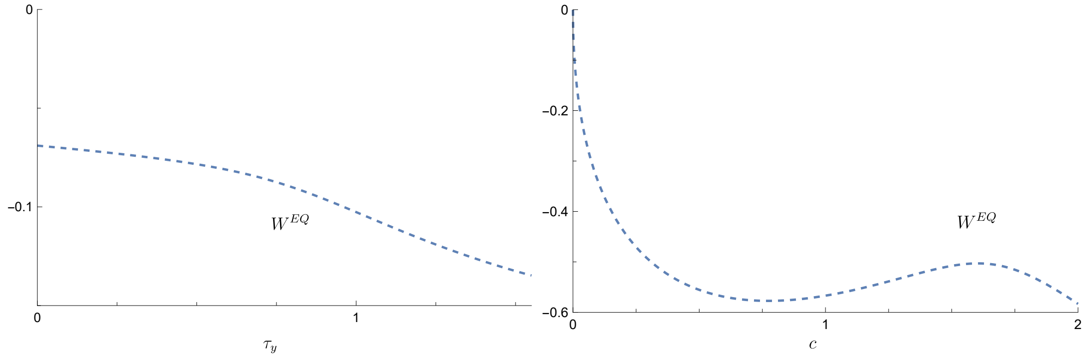

Figure 6: Counterintuitive comparative statics of welfare in τ_y (left) and c (right).

It might be surprising that transparency always increases welfare: After all, it has an ambiguous effect on total precision τ_Σ and other parameters may have perverse effects on welfare. In addition, inference from aggregative information is biased by cursedness while the fundamental sources, which generate these counterintuitive effects, are interpreted correctly. The key observation to understand this difference is that transparency renders the aggregative signal more informative relative to the fundamental sources of information, while the opposite is the case for the other parameters. Consequently, partially cursed agents substitute towards the aggregative signal, which ameliorates their bias. At worst, in the fully cursed case, the aggregative signal is not understood at all and hence irrelevant. However, as the fully cursed case makes apparent, there remain unreaped benefits from increased transparency in such economies.

7 Shrewd Agent: Behavior and Policy

In this section, we study the behavior and welfare of a *shrewd agent*: a fully rational, atomistic agent in the model, who understands its structure and is aware that all other agents (the *cursed crowd*) are χ-cursed. We discuss the results qualitatively in the text, relegating formal statements to Appendix C.

7.1 Best Response and Information Acquisition

We continue to denote the precision of information acquired by the cursed crowd as τ_s and denote the precision acquired by the shrewd agent as τ_s^R. The shrewd agent takes

only if $r > \frac{1}{2}$. Incidentally, this is the same threshold that Morris and Shin (2002) obtain for public information τ_y to be welfare reducing.

the equilibrium loadings (and information acquisition) of the cursed crowd as given, and chooses both how much private information to acquire as well as the coefficients in his linear action rule $a_i^R = \alpha_1^R s_i + \alpha_2^R y + \alpha_3^R p$. Formally, he solves

$$\max_{\alpha^R, \tau_s^R} W\left(\alpha^R, \delta, \tau_s^R\right) \tag{49}$$

Best responding to the equilibrium in the cursed crowd, the individual loadings of the shrewd agent, α^R, will differ from δ whenever $\chi > 0$. There is again a tight connection between the use of private information, α_1^R, and its acquisition through the envelope condition

$$\tau_s^R = \frac{\alpha_1^R}{\sqrt{c}}. \tag{50}$$

Denote the total precision available to the rational agent by $\tau_\Sigma^R := \tau_\theta + \tau_y + \tau_s^R + \delta_1^2 \tau_p$. We obtain an equation linking the information acquired by the shrewd agent and the cursed crowd

$$\frac{\tau_\Sigma}{\tau_\Sigma^R} = 1 + \frac{\chi \delta_1^2 \tau_P}{\tau_\theta + \tau_y + \tau_s}. \tag{51}$$

The shrewd agent acquires less information. Compared to the cursed crowd, he can substitute for it with a better comprehension of aggregative information. If the crowd is fully cursed, then (51) simplifies to

$$\tau_s^R = \tau_s - \delta_1^2 \tau_p \tag{52}$$

that is, the shrewd agent exactly offsets the information he can glean from the aggregative signal and his total precision is equal to the total precision *perceived* by the crowd.

Clearly, equation (52) holds only if it delivers a positive τ_s^R. Otherwise the shrewd agent will choose $\tau_s^R = 0$ as he is already satiated with the information he can infer from the aggregative signal. With a fully cursed crowd, this always happens with sufficiently large transparency since both τ_s and δ_1 are unresponsive to τ_p. In that case, transparency only serves as a cost-saving device for the shrewd agent.

The shrewd agent continues to free-ride on the crowd's use of private information even at interior levels of cursedness. He acquires a strictly positive amount of information if and only if

$$\tau_\theta + \tau_y \in \left(\frac{(1-r)\left(1 - \frac{1}{\tau_p \sqrt{c}}\right)}{\sqrt{c}}, \frac{1-r}{\sqrt{c}} \right) \tag{53}$$

Therefore, there is an inactivity region whenever $\tau_p > \frac{1}{\sqrt{c}}$; in that case, the shrewd agent acquires private information only if public information is sufficiently precise. The rationale is as follows: If public fundamental information is noisy the cursed crowd will acquire and use a lot of private information; since he can be parasitic on this information, the rational agent has no incentive to acquire information himself. As public information becomes more abundant, however, there is less information acquisition and use by the crowd. The aggregative source dries up and the shrewd agent needs to supplement it with private information acquisition. Finally, the upper bound on $\tau_\theta + \tau_y$ for the existence of a nontrivial equilibrium is the same for both classes of agents. In the trivial equilibrium $\delta_1 = \tau_s = 0$, the shrewd agent cannot utilize his comparative advantage in understanding the aggregative source since it is uninformative: he behaves identically to the crowd.

An immediate consequence of this inactivity region is that τ_s^R is nonmonotonic in τ_y. This contrasts with the unambiguously signed comparative statics for τ_s (Proposition 6). Similarly, the effect of information acquisition costs on τ_s^R is nonmonotonic and we can have an inactivity region (see Figure 7). Again, a change in parameters affects both the availability of aggregative information provided by the cursed crowd and the shrewd agent's demand for information overall.

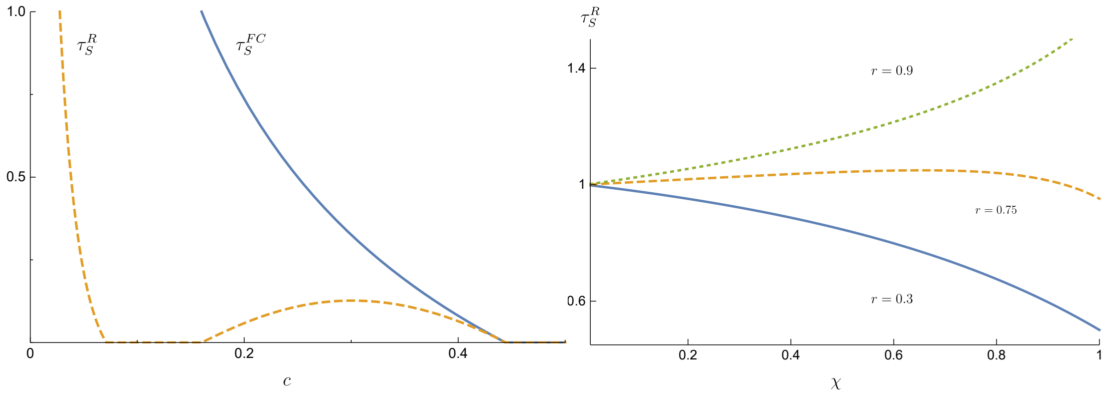

Figure 7: τ_s and τ_s^R as a function of c (left). τ_s^R (normalized to 1 at $\chi = 0$) as a function of χ for $r \in \{0.3, 0.75, 0.9\}$ (right).

The impact of cursedness on the precision of information acquired by the shrewd agent depends on the nature of strategic interactions (see Figure 7). Take as a benchmark the case of $r = 0$, i.e. all agents simply try to guess the true state, and all strategic interaction comes from the precision of aggregative information. In this case, τ_s^R is decreasing in χ as more cursed agents acquire and disseminate more information. Strategic substitutes increases this effect: As the crowd becomes more cursed, δ_1 increases which reduces the desire to match the state and therefore the value of private information. With complements, the opposite is the case: A higher δ_1 increases the desire to match θ and therefore – if this motive is sufficiently strong – information acquisition.

7.2 Welfare

Let W_χ^R denote the welfare of the shrewd agent facing an equilibrium δ^χ. Then,

$$W_\chi^{EQ} = W\left(\delta^\chi, \delta^\chi, \tau_s^\chi\right) \leq \max_{\alpha, \tau_s^R} W\left(\alpha, \delta^\chi, \tau_s^R\right) = W_\chi^R \tag{54}$$

As he comprehends his informational environment, he always obtains a higher welfare than the cursed crowd. The inequality is strict if $\chi > 0$.

We now ask whether the shrewd agent benefits from an increase in the cursedness of the crowd. This is the case for the first modicum of cursedness since for small positive ϵ

$$W_\epsilon^R > W_\epsilon^{EQ} > W_0^{EQ} = W_0^R. \tag{55}$$

The central inequality follows since in this region "cursedness is bliss" (Proposition 14).[29] In a highly cursed environment, however, the impact of cursedness depends on nature of the strategic interaction. If there are strategic substitutes, the shrewd agent always benefits from increased cursedness of the crowd: not only does he free-ride on aggregative information, but the crowd's over-reliance on the private signal helps him anti-coordinate. In the presence of complementarities, however, informational free-riding and the lack of coordination implied by cursed information misuse have opposing effects. While the shrewd agent can learn the state more precisely, his action has to follow the behavior of the less informed crowd. The latter effect can be overwhelming close to $\chi = 1$ so he would prefer an interior level of cursedness.

We conclude this section by studying the impact of precision and cost parameters on W_χ^R. If the crowd is close to rational, then policies have an impact similar to that in the rational equilibrium. We therefore focus our analysis on the other extreme case and evaluate the welfare of the shrewd agent facing a fully cursed crowd. By continuity, the results extend to a sufficiently cursed environment.

Recall that W_1^{EQ} is independent of τ_p as fully cursed agents do not respond to higher transparency. Therefore, an increase in transparency only affects the shrewd agent by providing a more precise aggregative signal, which is clearly beneficial.

Recall also that no paradoxical comparative statics in τ_y and c can emerge in a fully cursed environment: more public information and lower cost are always beneficial for the cursed crowd. There is a crowding out effect on the shrewd agent, however, as public fundamental information decreases information acquisition and dissemination by the cursed crowd. This is especially harmful if he is largely relying on this source of information, leading to the negative welfare impact when τ_s^R is small. Similarly, higher costs of information acquisition can be beneficial for the shrewd agent. This effect works through the action externality. Consider a situation where the shrewd agent does not acquire information himself – i.e. there is no direct effect of higher costs – and aggregate information is relatively abundant as τ_p is large. If costs are higher, cursed agents rely more on y. This makes it easier for the shrewd agent to anticoordinate with them, which is beneficial when $r < 0$ (strategic substitutes). This effect can dominate the harm from reduced information dissemination. If instead the shrewd agent is acquiring a positive amount of private information, these effects are dampened by the adjustment of τ_s^R and dominated by the direct impact of the change in acquisition costs.

The comparison between Proposition 15 and the results just sketched highlights qualitative differences in the impact of policy on the welfare of cursed and shrewd agents. Transparency leaves the welfare of the cursed crowd unaffected but has strictly positive (and

[29] Indeed, the information spillover can be strong enough to make the shrewd agent in the cursed world better off than first-best welfare (as can be checked for $r = 0$, $\tau_\theta = \tau_y = 0.1$, $\tau_p = 0.19$, $c = 0.03$, where we have $W_1^R > W^\star$) By continuity, this holds for an open set of parameters.

large) impact for the shrewd. In an augmented model where both types affect the aggregate outcome, this could easily turn into a redistribution result. This observation suggests that transparency can function as an elitist policy, giving an advantage to sophisticated agents who are able to understand and utilize aggregative information. For public information and lower costs this trade-off is already apparent in the present results.[30] A natural extension to study these questions would be a model of true cognitive heterogeneity featuring several non-atomistic groups with different levels of cursedness, all affecting the aggregate outcomes. Although the linear structure of the model makes action aggregation straightforward, the correlation between information use and acquisition affects the aggregate outcome and introduces nonlinearity. The analysis of such cognitive heterogeneity is therefore beyond the scope of this paper.

8 Conclusion

This paper studies the effect of aggregative information focusing on the interplay of two key aspects: First, that the precision of such aggregate statistics as signals of the fundamental depends on the amount of private information present in individual actions; and second, agents' well-documented difficulty in making inference based on such signals as it requires inferring others' information from their actions.

We conduct our analysis in a beauty contest game with information acquisition, adapting a notion of cursed equilibrium to model agents limited understanding of aggregative information. Though parsimonious, the model is sufficiently rich to relate to existing literature and offer alternative explanation of well-established phenomena such as the detrimental effect of public information and the irrelevance of transparency for informational efficiency. Since cursedness significantly alters the positive and normative results in our setting, it would be interesting to extend the analysis to more general payoff specifications as e.g. in Angeletos and Pavan (2007) and more deeply microfounded models yielding reduced forms similar to this class, as e.g. the business cycle model considered in Colombo et al. (2014) and demand function competition in Vives (2017).

We show that there is inefficiently low acquisition and use of private information in the rational benchmark due to an information dissemination externality. Cursed agents rely more heavily on their private information which can push information acquisition towards (or even above) its efficient level. While cursedness creates inefficiencies in information use, this effect initially dominates: a bit of individual cursedness is a collective blessing. Transparency crowds out the acquisition and use of private information but always increases the endogenous precision of the aggregative signal. This is the main driving force making it the only policy instrument with an unambiguously positive effect on welfare, despite its ambiguous effect on some measures of informational efficiency (Section 5) and its redistributive potential (Section 7).

Incorporating information acquisition into a model of incorrect information use, such as cursed equilibrium, is the main theoretical contribution of this paper. Doing so requires

[30] This conflict of interest between experts and unsophisticated actors casts doubt on the role of expert lobbying as a source of information on the impact of such policies.

making an assumption on how such agents asses the value of information. In our notion of *cursed expectations equilibrium with information acquisition*, agents correctly anticipate both the equilibrium strategies as well as how they will make use of their information, but they mistakenly consider this use to be optimal. This assumption is operationalized by a subjective envelope condition, which is highly tractable as it results in a close tie between information use and acquisition. While alternative notions do not conform to the behavioral desiderata in our setting, the properties and predictive power of such notions across applications of cursed equilibrium (and other behavioral equilibrium notions that do not easily allow a quasi-Bayesian analysis) remain an important question for future research.

A The Transparent Limit and the Price Paradox

An important special case is the limit as $\tau_p \to \infty$. In this *transparent limit*, agents observe not just a noisy signal, but the aggregate action itself and can condition their actions on it. In this case when the aggregative signal is noiseless, rational agents are able to infer the state. This generate the famous price paradox (Grossman and Stiglitz, 1980; Diamond and Verrecchia, 1981): Agents ignore their private information and cease to acquire any, thereby eliminating the very source of information in the aggregative signal. As a consequence, no equilibrium can exist.[31] We show that even an infinitesimal degree of cursedness is sufficient to restore existence when private information is exogenous: cursed agents fail to realize that the aggregative signal is fully revealing, and therefore continue to rely on their private information. Despite that, the value of private information shrinks to zero for agents that are approximately rational. For this reason, when information acquisition is costly an equilibrium in the transparent limit exists if and only if the degree of cursedness is sufficiently large.

We use the transparent limit to establish large τ_p results in the main text. Accordingly, the proofs pertaining to this appendix are given as part of Appendix B, where they flow with the formal development of our results.

A.1 The Transparent Limit with Exogenous τ_s

With fully rational agents, full transparency leads to the classic price paradox (Diamond and Verrecchia, 1981): Whenever $\delta_1 \neq 0$, knowledge of $\bar{a}$ translates into knowledge of the state which makes it suboptimal to place any weight on the private signal and hence $\delta_1 = \alpha_1 = 0$. If instead $\delta_1 = 0$, agents respond with a positive weight on their private signals $\alpha_1 > 0$, which is inconsistent. Hence, no equilibrium exists.

Cursed agents, however, continue to put a positive weight on the conditionally uninformative signals s_i, y even if they observe a fully informative aggregative signal. Even an infinitesimal amount of cursedness, therefore, resolves the price paradox since it re-introduces residual uncertainty about θ that was wiped out by infinite transparency.[32] Therefore,

Proposition 16. *An equilibrium of the limit game exists if and only if $\chi > 0$. It is given by*

$$\delta_1^\infty = \frac{\chi \tau_s (1-r)}{\tau_\theta + \tau_y + \tau_s (1 - r\chi)} \tag{56}$$

$$\delta_2^\infty = \frac{\chi^2 (1-r) \tau_y \tau_s}{\left(\tau_\theta + \tau_y + \tau_s (1-r\chi)\right)\left((1-\chi)\left(\tau_\theta + \tau_y\right) + (1-\chi r)\tau_s\right)} \tag{57}$$

$$\delta_3^\infty = \frac{(1-\chi)\left(\tau_\theta + \tau_y + \tau_s\right)}{(1-\chi)\left(\tau_\theta + \tau_y\right) + (1-\chi r)\tau_s} \tag{58}$$

Moreover, the equilibrium of the game with finite τ_p converges to δ^∞ as $\tau_p \to \infty$.

As $\chi \to 0$, we have an exclusive reliance on the fully revealing signal p, as $\delta^\infty \to (0,0,1)$. Close to the rational limit, agents also neglect y in favor of their private signal $\frac{\delta_2^\infty}{\delta_1^\infty} \to 0$, as all coordination can be done by loading on p. As cursedness increases, agents substitute away from this fully revealing source and towards noisy fundamental signals. As shown in Figure 8, substitution towards the private

[31] Vives (2014) shows that the price paradox can be solved even without introducing noise traders provided there is sufficient heterogeneity in traders' valuation; we show that (sufficient) cursedness also solves the paradox in a setting where agents are identical.

[32] Note that cursed agents also remain uncertain about $\bar{a}$: Despite being told its realization, they follow updating rule (10) which generates the posterior about $\bar{a}$ from the posterior about θ. Somewhat paradoxically, this consistency of posteriors about $(\bar{a}, \theta)$ delivers this unintuitive implication for posteriors on $(p, \bar{a})$.

signal dominates and coordination is still almost exclusively achieved through p at low degrees of cursedness, while the public signal gains importance at high degrees of cursedness.

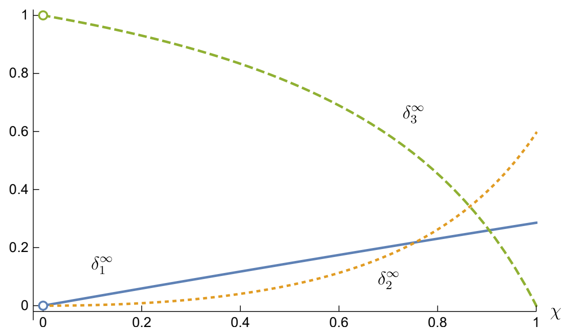

Figure 8: δ^∞ as a function of χ.

Restoring the existence of an equilibrium in the transparent limit reveals a key property of cursed equilibrium. Cursedness is different from dismissing part of the aggregative information, e.g. by scaling down its subjective precision. Indeed, such a scaling is powerless if the precision is infinite. Instead, cursed agents think that for any level of precision the information contained in the aggregative signal is not the whole story.

A.2 The Transparent Limit with Acquisition

With information acquisition, the optimality condition (33) implies that τ_s^∞ solves

$$\tau_s \sqrt{c} = \frac{\chi \tau_s (1-r)}{\tau_\theta + \tau_y + \tau_s (1 - r\chi)} \tag{59}$$

This equation always has a trivial solution with $\tau_s^\infty = 0$, though it constitutes an equilibrium only if condition (28) is violated. An interior limit equilibrium exists if

$$\chi > \sqrt{c} \frac{\tau_\theta + \tau_y}{(1-r)} \tag{60}$$

Contrary to the case without information acquisition, where an infinitesimal amount of cursedness was enough to overcome the price paradox and ensure existence, here a sufficiently large degree of cursedness is needed.[33] If condition (28) holds but (60) is violated, neither the trivial nor an interior equilibrium exists. Non-existence emerges in this setting because of information acquisition as in Grossman and Stiglitz (1980). Cursed agents are willing to use any private information they have, but the marginal value of acquiring it is smaller then its marginal cost unless condition (60) is satisfied.[34] Therefore they *do not acquire* (and a fortiori *cannot use*) any information, which is impossible in an informative equilibrium. On the contrary, the classic price paradox would persist even if private information were free as it would remain *unused* due to the abundance of aggregative information, which is impossible in equilibrium.

[33] Condition (60) does not contradict the sufficiency of condition (28) which holds for any interior τ_p. Here, we are in a situation where $\tau_p = \infty$. The order of limits is relevant, as for every fixed τ_p, the influence of transparency vanishes as δ_1 and τ_s go to zero.

[34] Indeed, existence of the transparent limit is guaranteed if the acquisition cost satisfies an Inada-type condition at zero as the generic condition reads $c'(0) < \left(\frac{\chi(1-r)}{\tau_\theta + \tau_y}\right)^2$.

Proposition 17. *There exists a nontrivial equilibrium in the transparent limit if and only if* $\chi > \sqrt{c}\frac{\tau_\theta+\tau_y}{(1-r)}$. *It is given by (31)-(33) with*

$$\delta_1^\infty = \frac{\chi(1-r) - \left(\tau_\theta + \tau_y\right)\sqrt{c}}{(1-r\chi)} \tag{61}$$

In Section A.1, we have seen that in the transparent limit with exogenous τ_s, agents rely almost exclusively on aggregative information as $\chi \to 0$. Close to rationality, we now have an existence problem. However, for χ converging to its lower bound $\sqrt{c}\frac{\tau_\theta+\tau_y}{(1-r)}$, we again have an almost exclusive reliance on the aggregative signal, $\delta^\infty \to (0,0,1)$. Again, as cursedness increases, the reliance on the

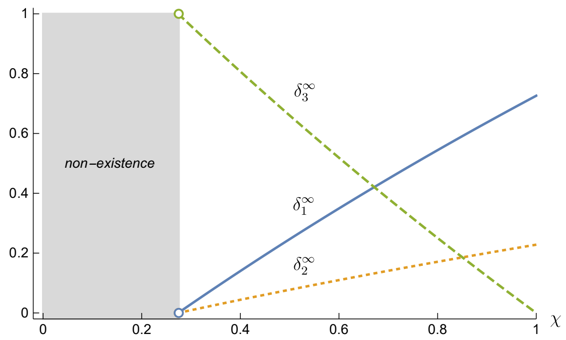

Figure 9: δ^∞ with information acquisition as a function of χ.

aggregative signal vanishes and the loadings on the other two signals increase. Comparing Figure 9 to the case without information acquisition (Fig. 8), however, we see that there is no crossing between δ_1, δ_2 as their ratio is constant in χ (it can be larger or smaller than one, depending on other parameters).

Remark (Welfare in the Transparent Limit). Contrary to the efficient solution, which does not admit a limit as $\tau_p \to \infty$, the welfare formula (40) remains valid. In the transparent limit, equilibrium welfare is given by

$$W^{EQ} = -\frac{2\sqrt{c}\chi(1-r)^2 - c(1-2r+r\chi)\left(\tau_\theta+\tau_y\right)}{(1-r)(1-\chi r)} \tag{62}$$

It is easy to show that $\frac{\partial W^{EQ}}{\partial \chi} < 0$, seemingly overturning the result that cursedness is bliss. This is not the case since the bliss result holds close to $\chi = 0$ but the transparent limit equilibrium only exists for χ sufficiently large.

B Proofs

B.1 Proofs for Section 3 (Model with Fixed τ_s)

Proof of Proposition 1: Recall from the text that $\delta_i = \alpha_i$ and the best response

$$a_i = (1-r)\left(\chi\frac{\tau_s s_i + \tau_y y}{\tau_\theta+\tau_y+\tau_s} + (1-\chi)\frac{\tau_s s_i + \tau_y y + \delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3}y\right]}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right)$$
$$+ r\left(\alpha_0 + \alpha_1\left(\chi\frac{\tau_s s_i + \tau_y y}{\tau_\theta+\tau_y+\tau_s} + (1-\chi)\frac{\tau_s s_i + \tau_y y + \delta_1^2\tau_p\frac{1-\delta_3}{\delta_1}\left[p - \frac{\delta_2}{1-\delta_3}y\right]}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right) + \alpha_2 y + \alpha_3 p\right)$$

$$a_i = \alpha_0 + \alpha_1 s_i + \alpha_2 y + \alpha_3 p$$

Matching coefficients, we get

$$\delta_0 = r\delta_0 \tag{63}$$

$$\delta_1 = (1 - r + r\delta_1)\left(\frac{\tau_s\chi}{\tau_\theta + \tau_y + \tau_s} + \frac{\tau_s(1-\chi)}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) \tag{64}$$

$$\delta_2 = (1 - r + r\delta_1)\left(\chi\frac{\tau_y}{\tau_\theta + \tau_y + \tau_s} + (1-\chi)\frac{\tau_y - \delta_1\delta_2\tau_p}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right) + r\delta_2 \tag{65}$$

$$\delta_3 = (1-\chi)\left[(1-r) + r\delta_1\right]\frac{\delta_1\tau_p(1-\delta_3)}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p} + r\delta_3 \tag{66}$$

It is easy to see that $\delta_0 = 0$ since $r < 1$. Given δ_1, the latter two equations are linear and we can solve for

$$\delta_2 = \frac{(1 - r + r\delta_1)\left(\chi\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s} + (1-\chi)\frac{\tau_y}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}\right)}{1 - r + (1 - r + r\delta_1)(1-\chi)\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}} \tag{67}$$

$$\delta_3 = \frac{(1-\chi)\left[(1-r) + r\delta_1\right]\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}}{(1-r) + (1-\chi)\left[(1-r) + r\delta_1\right]\frac{\delta_1\tau_p}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}}. \tag{68}$$

Reformulating the equation for δ_1,

$$\delta_1 = \left[1 - r + r\delta_1\right]\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\left(1 + \chi\frac{\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s}\right)$$

We arrive at the cubic equation f from

$$\begin{aligned}
\delta_1\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right) &= \left[1 - r + r\delta_1\right]\tau_s\left(1 + \chi\frac{\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s}\right) \\
0 = f(\delta_1) &:= \delta_1\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}\left[1 - r + r\delta_1\right]\delta_1^2\tau_p - (1-r)(1-\delta_1)\tau_s \qquad (69) \\
&= \delta_1\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right) - \tau_s\left[1 - r + r\delta_1\right]\left\{1 + \chi\frac{\delta_1^2\tau_p}{\tau_\theta + \tau_y + \tau_s}\right\}
\end{aligned}$$

To show that the solution is unique, note that $\frac{f(\delta_1)}{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}$ evaluated at $\delta_1 = 0$ is equal to $-\frac{1-r}{\tau_\theta+\tau_y+\tau_s} < 0$ and at $\delta_1 = 1$ it is greater than $1 - \frac{\tau_s}{\tau_\theta+\tau_y+\tau_s} > 0$. Hence, there is at least one root in $(0,1)$. Furthermore, the expression is increasing at a root, as

$$\frac{\partial}{\partial\delta_1}\frac{f(\delta_1)}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p} = 1 - r\tau_s\left[\frac{\chi}{\tau_\theta + \tau_y + \tau_s} + \frac{1-\chi}{\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p}\right] + 2(1-\chi)\delta_1\tau_p\tau_s\frac{1 - r + r\delta_1}{\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)^2}$$

where the first to terms are in sum positive and, at a root, we have $\operatorname{sgn}\{\delta_1\left[(1-r) + r\delta_1\right]\} = 1$ whence the final term is also positive.

As a corollary of this argument, we obtain

Corollary 1. *In equilibrium, we have* $(1-r) + r\delta_1 > 0$.

Using the rewriting of f

$$[(1-r)+r\delta_1] = \frac{\delta_1}{\tau_s \frac{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}} \tag{70}$$

to express $(1-r)+r\delta_1$ we get

$$\delta_2 = \frac{\delta_1\tau_y\left(\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p\right)}{(1-r)\tau_s\left(\tau_\theta+\tau_y+\tau_s\right)+\delta_1^2\tau_p\left((1-\chi)\left(\tau_\theta+\tau_y\right)+(1-r\chi)\tau_s\right)} \tag{71}$$

$$\delta_3 = \frac{(1-\chi)\delta_1^2\tau_p}{(1-r)\tau_s+\delta_1^2\tau_p\left(1-\chi+(1-r)\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\right)} \tag{72}$$

Note that from (71) and (72), we have $\delta_2 > 0$ and $\delta_3 \geq 0$. Solving for χ from (64) we get

$$\chi = \frac{\delta_1\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-(1-r)(1-\delta_1)\tau_s}{\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}[1+r(\delta_1-1)]\delta_1^2\tau_p}$$

and plugging this into the above δ_2, δ_3, we obtain the desired expressions. □

Proof of Proposition 16: From the equilibrium condition (69) we see that

$$\delta_1^2\tau_p\left(\delta_1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}[(1-r)+r\delta_1]\right)+\delta_1\left(\tau_\theta+\tau_y\right)-(1-r)(1-\delta_1)\tau_s = 0$$

so for $\delta_1^2\tau_p$ to go unbounded, we need that the parenthesis goes to zero:

$$\delta_1 = \frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}$$

for some constant ξ. Plugging this expression into f, we get

$$\xi+\left(\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}\right)\left(\tau_\theta+\tau_y\right)-(1-r)\left(1-\left(\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}+\frac{\xi}{\delta_1^2\tau_p}\right)\right)\tau_s = 0$$

$$\xi = (1-r)\frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left(\tau_\theta+\tau_y\right)-\left(1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\right)\tau_s}{\left(1+\frac{\tau_\theta+\tau_y+(1-r)\tau_s}{\delta_1^2\tau_p}\right)\left(1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\right)}$$

So we just constructed a solution that converges for $\chi > 0$. So, in the transparent limit, we have

$$\delta_1^\infty = \frac{\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)}{1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r}$$

To arrive at δ_2^∞ and δ_3^∞,we plug δ_1^∞ into (13) and (14). It is easy to see that $\frac{d\delta_2^\infty}{d\chi} \geq 0$. The existence of a limit equilibrium follows, as all the arguments leading to equilibrium are well defined even taking $\tau_p \to \infty$ assuming $\delta_1 > 0$, which is the case in the solution if and only if $\chi > 0$. □

Proof of Proposition 2: The result follows from implicit differentiation of the equilibrium equation (69). To this purpose, let us first establish a helpful lemma.

Lemma 1. *In equilibrium, we have $f_\delta > 0$.*

Proof of Lemma: Compute

$$
\begin{aligned}
f_\delta &= \tau_\theta + \tau_y + 3\delta_1^2\tau_p - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2 + r(3\delta_1 - 2)\right]\delta_1\tau_p + (1-r)\tau_s \\
&= \left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\left[1 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[\frac{1+r(\delta_1-1)}{\delta_1}\right]\right]\right) + \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1-r\right]\delta_1\tau_p + (1-r)\tau_s \\
&= \left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\left[1 - \frac{\chi\tau_\theta+\chi\tau_y+\chi\tau_s+\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right]\right) + \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1-r\right]\delta_1\tau_p + (1-r)\tau_s > 0
\end{aligned}
$$

where in the final step we used (70) in the transformation:

$$
1 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[\frac{1+r(\delta_1-1)}{\delta_1}\right] = 1 - \frac{\chi\tau_\theta+\chi\tau_y+\chi\tau_s+\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p} \geq 0. \qquad \blacktriangle
$$

By implicit differentiation

$$
\frac{d\delta_1}{d\chi} = -\frac{f_\chi}{f_\delta} \propto \frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r) + r\delta_1\right]\delta_1^2\tau_p \geq 0
$$

and from (13), (14) it is immediate that $\frac{d\delta_2}{d\chi} \propto \frac{d\delta_1}{d\chi} \geq 0$ and $\frac{d\delta_3}{d\chi} \propto -\frac{d\delta_1}{d\chi} \leq 0$.

Relative size of δ_1 and δ_2:

$$
\frac{d}{d\chi}\frac{\delta_2}{\delta_1} = \frac{d}{d\chi}\frac{\delta_1\tau_y}{(1-r)\tau_s - \delta_1\left(\tau_\theta+\tau_y\right)} = \frac{\tau_y\left((1-r)\tau_s - \delta_1\left(\tau_\theta+\tau_y\right)\right) + \delta_1\tau_y\left(\tau_\theta+\tau_y\right)}{\left((1-r)\tau_s - \delta_1\left(\tau_\theta+\tau_y\right)\right)^2}\frac{d\delta_1}{d\chi} \geq 0
$$

whence the result in the proposition follows.

τ_p comparative statics: Again, $\frac{d\delta_1}{d\tau_p} \propto -f_{\tau_p}$ and using (70), we have

$$
f_{\tau_p} = \delta_1^3 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r) + r\delta_1\right]\delta_1^2 = \delta_1^3\left(1 - \chi\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right) > 0
$$

From (13) and (14), the comparative statics are immediate. Finally, we have

$$
\begin{aligned}
\frac{d\delta_1^2\tau_p}{d\tau_p} &= 2\delta_1\tau_p\frac{d\delta_1}{d\tau_p} + \delta_1^2 \\
&= -2\delta_1\tau_p\frac{\delta_1^3 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\delta_1^2}{\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[2+r(3\delta_1-2)\right]\delta_1\tau_p + (1-r)\tau_s} + \delta_1^2 \\
&= \frac{\delta_1^2}{f_\delta}\left\{-2\delta_1\tau_p\left(\delta_1 - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[(1-r)+r\delta_1\right]\right) + \left(\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right) - \chi\frac{\tau_s\left[2+r(3\delta_1-2)\right]}{\tau_\theta+\tau_y+\tau_s}\delta_1\tau_p + (1-r)\tau_s\right)\right\} \\
&\propto \left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\delta_1^2\tau_p + (1-r)\tau_s \\
&= \frac{1}{\delta_1}\left((1-r)\tau_s + \chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}(1-r)\delta_1^2\tau_p\right) > 0
\end{aligned}
$$

where we dropped $\frac{\delta_1^2}{f_\delta} > 0$ and, in the last step, we use $f(\delta_1) = 0$, which establishes the claim. □

Proof of Proposition 3: Since we have $f_\delta > 0$ (Lemma 1), we get for a generic parameter v

$$\frac{\mathrm{d}\delta_1}{\mathrm{d}v} = -\frac{f_v}{f_\delta} \propto -f_v$$

And hence the comparative statics of δ_1 follow immediately, occasionally using (70), from

$$\begin{aligned}
f_{\tau_y} &= \delta_1 + \chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p\frac{\tau_s}{\left(\tau_\theta+\tau_y+\tau_s\right)^2} > 0\\
f_{\tau_s} &= -\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p - (1-r)(1-\delta_1) < 0\\
f_r &= \chi\frac{\tau_s(1-\delta_1)}{\tau_\theta+\tau_y+\tau_s}\delta_1^2\tau_p + (1-\delta_1)\tau_s = \tau_s(1-\delta_1)\left[\frac{\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s}+1\right] > 0
\end{aligned}$$

□

Proof of Proposition 4: The result for $\frac{d\beta}{d\chi}, \frac{d\beta}{d\tau_p}, \frac{d\gamma_2}{d\tau_p}$ is immediate from the comparative statics of δ_1. For $\frac{d\gamma_2}{d\tau_y} \propto \frac{d\delta_1}{d\tau_y}\tau_y + \delta_1$, we get

$$\begin{aligned}
\frac{d\delta_1}{d\tau_y}\tau_y + \delta_1 &= -\frac{f_{\tau_y}}{f_\delta}\tau_y + \delta_1 \propto -f_{\tau_y}\tau_y + f_\delta\delta_1\\
&= \frac{\delta_1}{\left(\tau_y+\tau_s+\tau_\theta\right)^2}\left[\left(3\delta_1^2\tau_p+\tau_\theta+(1-r)\tau_s\right)\left(\tau_y+\tau_s+\tau_\theta\right)^2 - \chi\delta_1\tau_p\tau_s\left((3-3r+4r\delta_1)\tau_y+(2-2r+3r\delta_1)(\tau_s+\tau_\theta)\right)\right]\\
&\propto \underbrace{\left(3\delta_1^2\tau_p+\tau_\theta+(1-r)\tau_s\right)\left(\tau_y+\tau_s+\tau_\theta\right)^2 - 3\chi\delta_1\tau_p\tau_s(1-r+r\delta_1)\left(\tau_y+\tau_s+\tau_\theta\right)}_{=:\mathrm{A}_1} + \mathrm{A}_2
\end{aligned}$$

where, using (70),

$$\mathrm{A}_1 = \left(\tau_y+\tau_s+\tau_\theta\right)^2\left[\tau_\theta+(1-r)\tau_s+3\frac{(1-\chi)\delta_1^2\tau_p\left(\tau_y+\tau_s+\tau_\theta\right)}{\tau_y+\tau_s+\tau_\theta+\chi\delta_1^2\tau_p}\right] > 0.$$

It remains to show that $\mathrm{A}_2 > 0$. We have

$$\begin{aligned}
\mathrm{A}_2 &\propto (1-r)(\tau_\theta+\tau_s) - \tau_y r\delta_1 > (1-r)(\tau_\theta+\tau_s) - \tau_y r\delta_1^{\mathrm{FC}}\\
&= (1-r)(\tau_\theta+\tau_s) - \tau_y r\frac{(1-r)\tau_s}{\tau_\theta+\tau_y+(1-r)\tau_s} = (1-r)\left(\tau_\theta+\tau_s\left(1-r\frac{\tau_y}{\tau_\theta+\tau_y+(1-r)\tau_s}\right)\right) > 0.
\end{aligned}$$

Note that the result for $\frac{d\gamma_2}{d\tau_s}$ and $\frac{d\beta}{d\tau_s}$ follows when we establish $\frac{d}{d\tau_s}\frac{\delta_1}{\tau_s} < 0$:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}\tau_s}\frac{\delta_1}{\tau_s} &= \frac{\frac{d\delta_1}{d\tau_s}\tau_s - \delta_1}{\tau_s^2} \propto \frac{d\delta_1}{d\tau_s}\tau_s - \delta_1 \propto -f_{\tau_s}\tau_s + f_\delta\delta_1\\
&= (1-r)(1-\delta_1)\tau_s - \delta_1\left(\tau_y+\tau_\theta+(1-r)\tau_s\right) + 3\frac{\chi(1-r)\tau_s\delta_1^2\tau_p}{\tau_y+\tau_\theta+\tau_s} - \frac{\chi(1-r)\tau_s^2\delta_1^2\tau_p}{\left(\tau_y+\tau_\theta+\tau_s\right)^2}\\
&\quad - \delta_1^3\tau_p\left(3-4\frac{\chi r\tau_s}{\tau_y+\tau_\theta+\tau_s}+\frac{\chi r\tau_s^2}{\left(\tau_y+\tau_\theta+\tau_s\right)^2}\right)
\end{aligned}$$

using f to replace $(1-r)(1-\delta_1)\tau_s$, we obtain a decomposition $\mathrm{B}_1+\mathrm{B}_2$ where

$$\mathrm{B}_1 = \frac{\chi\delta_1^2\tau_p\tau_s}{\left(\tau_y+\tau_\theta+\tau_s\right)^2}\left[\delta_1 r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s\right]$$
$$\mathrm{B}_2 = -\delta_1\left(2\delta_1^2\tau_p+(1-r)\tau_s\right)+2\frac{\chi\delta_1^2\left[1-r+\delta_1 r\right]\tau_p\tau_s}{\tau_y+\tau_\theta+\tau_s}$$

Note that in B_1 the final term is negative if $r<0$, otherwise, estimate $\delta_1<\delta_1^{\mathrm{FC}}$ to arrive at

$$\delta_1 r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s < \frac{(1-r)\tau_s}{\tau_\theta+\tau_y+(1-r)\tau_s}r\left(\tau_y+\tau_\theta\right)-(1-r)\tau_s$$
$$= -\frac{(1-r)^2\tau_s\left(\tau_y+\tau_\theta+\tau_s\right)}{\tau_\theta+\tau_y+(1-r)\tau_s}<0$$

and therefore $\mathrm{B}_1<0$. Plugging (70) into B_2, we arrive at

$$\mathrm{B}_2 = -\delta_1\left((1-r)\tau_s+2\frac{(1-\chi)\delta_1^2\tau_p\left(\tau_y+\tau_\theta+\tau_s\right)}{\left(\tau_y+\tau_\theta+\tau_s+\chi\delta_1^2\tau_p\right)}\right)<0,$$

whence we have established $\frac{d}{d\tau_s}\frac{\delta_1}{\tau_s}<0$ and therefore $\frac{d\gamma_2}{d\tau_s}<0$ and $\frac{d\beta}{d\tau_s}>0$. □

B.2 Proofs for Section 4 & 5 (Model with Information Acquisition)

Proof of Proposition 5: Equation (33) is derived in the text assuming that $\delta_1\geq 0$. We now show that there cannot be an equilibrium with $\delta_1<0$.

Lemma 2. *There is no equilibrium with information acquisition and $\delta_1<0$.*

Proof of Lemma: Towards a contradiction, let $\delta_1<0$. Then, $\tau_s=-\frac{\delta_1}{\sqrt{c}}$ and f reads

$$\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)+\chi\frac{1}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p+\frac{1}{\sqrt{c}}(1-r)(1-\delta_1)=0$$

Clearly, for $\delta_1=0$, the expression is strictly positive. Furthermore, we have $\frac{\mathrm{d}}{\mathrm{d}\delta_1}f<0$, as

$$2\delta_1\tau_p+\chi\frac{\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p}{\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2}+\chi\frac{\left[(1-r)+3r\delta_1^2\right]\tau_p}{\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1}-\frac{1}{\sqrt{c}}(1-r)=$$
$$\frac{1}{\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2}\Big[2\delta_1\tau_p\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2+\chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p$$
$$+\chi\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)\left[2(1-r)\delta_1+3r\delta_1^2\right]\tau_p-\frac{1}{\sqrt{c}}(1-r)\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2\Big]=$$
$$\propto 2\delta_1^3\tau_p-4\delta_1\tau_p\sqrt{c}\left(\tau_\theta+\tau_y\right)+2\delta_1\tau_p c\left(\tau_\theta+\tau_y\right)^2+\chi\left[(1-r)+r\delta_1\right]\delta_1^2\tau_p$$
$$+\chi\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)\left[2(1-r)\delta_1+3r\delta_1^2\right]\tau_p-\frac{1}{\sqrt{c}}(1-r)\left(\sqrt{c}\left(\tau_\theta+\tau_y\right)-\delta_1\right)^2=$$
$$=2\delta_1^3\tau_p(1-\chi r)-\delta_1^2\left[\frac{1}{\sqrt{c}}(1-r)+\left(4\tau_p-3\chi r\tau_p\right)\sqrt{c}\left(\tau_\theta+\tau_y\right)+\chi(1-r)\tau_p\right]$$
$$+\delta_1 2c\left(\tau_\theta+\tau_y\right)\left[(1-r)\left(1+\chi\tau_p\sqrt{c}\right)\tau_p+\tau_p c\left(\tau_\theta+\tau_y\right)\right]-\sqrt{c}(1-r)\left(\tau_\theta+\tau_y\right)^2<0$$

Hence, we cannot have an solution as $f > 0$ for all $\delta_1 < 0$ and there is no such equilibrium. □

Equations (29)-(32) follow immediately from plugging (33) into (11)-(14). The equilibrium δ_1 solves $0 = f(\delta_1, \frac{\delta_1}{\sqrt{c}})$, where

$$f(\delta_1, \frac{\delta_1}{\sqrt{c}}) = \delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) - [1 - r + r\delta_1]\left(1 + \chi\frac{\sqrt{c}\delta_1^2\tau_p}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}\right)$$

In equilibrium, using (70) and (33), we have

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\delta_1} f(\delta_1, \frac{\delta_1}{\sqrt{c}}) &= \frac{(1-r)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right) + \sqrt{c}\delta_1^2\tau_p\chi\left[\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)} - r\right]}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)} + 2\sqrt{c}\delta_1\tau_p\left((1-\chi)\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)}\right)\\ &\geq \frac{(1-r)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right) + \sqrt{c}\delta_1^2\tau_p\chi\left[\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)} - r\right]}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)} \geq \frac{\sqrt{c}\delta_1^2\tau_p\chi[1-r]}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)} > 0\end{aligned}$$

since the first term in the numerator is positive and the fraction in square brackets is greater than 1.

If we are in a corner case, we obtain δ_2, δ_3 by plugging $\delta_1 = \tau_s = 0$ to the original matching coefficients equations (67) and (68) to obtain $\delta_2 = \frac{\tau_y}{\tau_\theta + \tau_y}$ and $\delta_3 = 0$. □

Proof of Proposition 17: In the text. □

Proof of Proposition 6: Our system is defined by

$$\begin{aligned}f(\delta_1, \tau_s) &= \delta_1\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}[1 + r(\delta_1 - 1)]\delta_1^2\tau_p - (1-r)(1-\delta_1)\tau_s = 0\\ g(\delta_1, \tau_s) &= \delta_1^2 - c(\tau_s)^2 = 0\end{aligned}$$

and by implicit differentiation,

$$\frac{\mathrm{d}\delta_1}{\mathrm{d}v} = \frac{g_{\tau_s} f_v - f_{\tau_s} g_v}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta}, \quad \frac{\mathrm{d}\tau_s}{\mathrm{d}v} = \frac{f_\delta g_v - g_\delta f_v}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta}$$

First, we establish that the denominator of our implicit derivatives is positive.

Lemma 3. *In equilibrium, we have $g_\delta f_{\tau_s} - g_{\tau_s} f_\delta > 0$, and hence $\frac{\mathrm{d}\delta_1}{\mathrm{d}v} \propto g_{\tau_s} f_v - f_{\tau_s} g_v$.*

Proof of Lemma: Note that $g_\delta = 2\delta_1 > 0$, $g_{\tau_s} = -2c\tau_s < 0$ and

$$\begin{aligned}f_\delta &= \left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}[2 + r(3\delta_1 - 2)]\delta_1\tau_p + (1-r)\tau_s\\ f_{\tau_s} &= -\chi\frac{\tau_\theta + \tau_y}{\left(\tau_\theta + \tau_y + \tau_s\right)^2}[1 + r(\delta_1 - 1)]\delta_1^2\tau_p - (1-r)(1-\delta_1).\end{aligned}$$

By direct computation

$$\begin{aligned}g_\delta f_{\tau_s} - g_{\tau_s} f_\delta =& 2\delta_1\left(-\chi\frac{\tau_\theta + \tau_y}{\left(\tau_\theta + \tau_y + \tau_s\right)^2}[1 + r(\delta_1 - 1)]\delta_1^2\tau_p - (1-r)(1-\delta_1)\right)\\ &- (-2c\tau_s)\left(\left(\tau_\theta + \tau_y + 3\delta_1^2\tau_p\right) - \chi\frac{\tau_s}{\tau_\theta + \tau_y + \tau_s}[2 + r(3\delta_1 - 2)]\delta_1\tau_p + (1-r)\tau_s\right)\end{aligned}$$

$$
\begin{aligned}
=&2\delta_1\left(-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-\left[1+r(\delta_1-1)\right]+\delta_1\right)\\
&+2\frac{\delta_1}{\tau_s}\left(\left(\tau_\theta+\tau_y+3\delta_1^2\tau_p\right)\delta_1-2\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p+(1-r)\tau_s\delta_1-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\delta_1\delta_1^2\tau_p\right)\\
\overset{(f_\delta>0)}{\geq}&2\frac{\delta_1}{\tau_s}\left(-\chi\frac{\tau_\theta+\tau_y}{\tau_\theta+\tau_y+\tau_s}\frac{\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)\delta_1}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\delta_1^2\tau_p-\frac{\delta_1\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}+\delta_1\tau_s\right)\\
&+2\frac{\delta_1}{\tau_s}\left(\left(\tau_\theta+\tau_y+3\left(1-\chi\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right)\delta_1^2\tau_p\right)\delta_1+(1-r)\tau_s\delta_1+\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1-r\right]\delta_1^2\tau_p\right)\\
=&2\delta_1^2\left\{-\frac{\tau_y}{\tau_s}-\frac{\tau_\theta}{\tau_s}+\delta_1^2\tau_p\left(-\frac{1}{\tau_s}+\frac{1}{\tau_\theta+\tau_y+\tau_s}-\frac{1-\chi}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right)\right\}\\
&+2\frac{\delta_1}{\tau_s}\left(\left(\tau_\theta+\tau_y+3\left(1-\chi\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right)\delta_1^2\tau_p\right)\delta_1+(1-r)\tau_s\delta_1+\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1-r\right]\delta_1^2\tau_p\right)\\
\overset{(r<1\&(33))}{\geq}&2\delta_1^3\tau_p(1-\chi)\sqrt{c}\left\{\frac{2\delta_1^2+\sqrt{c}\chi\delta_1^3\tau_p+4\sqrt{c}\delta_1\left(\tau_\theta+\tau_y\right)+2c\left(\tau_\theta+\tau_y\right)^2}{\left[\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right]\left[\delta_1+\sqrt{c}\chi\delta_1^2\tau_p+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right]}\right\}\geq 0
\end{aligned}
$$

The last equality follows from lengthy but straightforward calculation. The inequality follows since the expression is decreasing in r and we hence set $r=1$ as a worst case, obtaining our result. ▲

Hence, we have

$$
\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}\propto g_{\tau_s}f_\chi-f_{\tau_s}g_\chi=g_{\tau_s}f_\chi=\left(-c''(\tau_s)(\tau_s)^2-2c'(\tau_s)\tau_s\right)\left(-\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p\right)>0.
$$

For $\frac{d\delta_2}{d\chi}$, using (31) $\frac{\mathrm{d}\delta_2}{\mathrm{d}\chi}=\frac{\sqrt{c}\tau_y}{(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}>0$. Finally, from (32) $\frac{\mathrm{d}\delta_3}{\mathrm{d}\chi}=-\frac{(1-r)}{(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)}\frac{\mathrm{d}\delta_1}{\mathrm{d}\chi}<0$.

For the remaining comparative statics let us begin with δ_1. Note that that for all precision parameters, we have $g_\tau\equiv 0$, and for costs, we have $f_c\equiv 0$. For the cost parameter we have $\frac{\mathrm{d}\delta_1}{\mathrm{d}c}\propto f_{\tau_s}(\tau_s)^2<0$ as $f_{\tau_s}<0$. For τ_s, we get $\frac{\partial\tau_s}{\partial c}=\frac{1}{c}\frac{\partial\delta_1}{\partial c}-\tau_s c<0$. We have $\frac{\partial\delta_1}{\partial\tau_y}\propto g_{\tau_s}f_{\tau_y}<0$ since $f_{\tau_y}>0$ and $\frac{\partial\delta_1}{\partial\tau_\theta}=\frac{\partial\delta_1}{\partial\tau_y}<0$. For τ_p, we have $\frac{\partial\delta_1}{\partial\tau_p}\propto g_{\tau_s}f_{\tau_p}<0$ since $f_{\tau_p}>0$. For r, we get $\frac{\partial\delta_1}{\partial r}\propto g_{\tau_s}f_r<0$ since $f_r\geq 0$.

For δ_2, the comparative static wrt τ_p is immediate.

To see the ambiguity of $\frac{d\delta_2}{d\tau_y}$, consider the transparent limit expression and take the derivative to obtain

$$
\begin{aligned}
\frac{d\delta_2^\infty}{d\tau_y}=&\frac{\sqrt{c}\left[\chi(1-r)-\sqrt{c}\left(2\tau_\theta+\tau_y\right)\right](1-r\chi)\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)+\sqrt{c}\tau_y\left[\chi(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right](1-r\chi)\sqrt{c}}{\left[(1-r\chi)\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\right]^2}\\
&\propto\left[\chi(1-r)-\sqrt{c}\left(2\tau_\theta+\tau_y\right)\right]\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)+\tau_y\left[\chi(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right]\sqrt{c}
\end{aligned}
$$

At $\chi=1$, we get $\frac{d\delta_2^\infty}{d\tau_y}\propto\left[(1-r)-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right]^2>0$,whereas at the lower limit $\chi=\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r}$, we have $\frac{d\delta_2^\infty}{d\tau_y}\propto-\sqrt{c}\tau_y\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)<0$, establishing the claim.

To see the other ambiguous comparative statics, we proceed in a similar fashion. Consider $\frac{d\delta_2}{dc}$; in the transparent limit, for $\chi=1$ we get $\frac{d\delta_2^\infty}{dc}=\frac{\tau_y}{2\sqrt{c}(1-r)}>0$, while at the lower bound, $\chi\to\sqrt{c}\frac{\tau_\theta+\tau_y}{1-r}$ we have $\frac{d\delta_2^\infty}{dc}\propto-(1-r)\tau_y\left(\tau_\theta+\tau_y\right)<0$. For $\frac{d\delta_2}{dr}$; in the transparent limit, for $\chi=1$ we get $\frac{d\delta_2^\infty}{dc}=\frac{\sqrt{c}\tau_y}{(1-r)^2}>0$, while at the lower bound, $\chi\to\sqrt{c}\frac{\tau_\theta+\tau_y}{1-r}$ we have $\frac{d\delta_2^\infty}{dr}\propto-c\tau_y\left(\tau_\theta+\tau_y\right)<0$.

For δ_3, the τ_p comparative static is immediate. To see the ambiguous comparative statics in τ_y, consider

$$\lim_{\chi\to 1}\frac{\frac{\partial\delta_3}{\partial\tau_y}}{\delta_3}\propto\frac{\left(1-\delta_1(1-r)-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}{1-\frac{\delta_1(1-r)}{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)}}=1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)>0$$

proving $\frac{\partial\delta_3}{\partial\tau_y}$ converges to 0 from above as $\chi\to 1$. Therefore, δ_3 is increasing in τ_y for large χ. However, consider the limit as $r\to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$, then

$$\frac{\partial\delta_3}{\partial\tau_y}\to-\frac{1-r}{\tau_\theta+\tau_y}\delta_1<0$$

To see that $\frac{\partial\delta_3}{\partial c}$ is of ambiguous sign, consider the limit as $\sqrt{c}\to\frac{1-r}{\tau_\theta+\tau_y}$. Then, we have

$$\operatorname{sgn}\left\{\frac{\partial\delta_3}{\partial c}\right\}\to\operatorname{sgn}\left\{-(1-r)^3\right\}<0$$

Furthermore, as $c\to 0$, we have

$$\frac{\partial\delta_3}{\partial c}\propto\delta_1^2(1-r)\left((1-\delta_1)\left(\tau_\theta+\tau_y\right)+(1-\chi)\delta_1^2\tau_p\right)>0$$

Following similar arguments, $\frac{\partial\delta_3}{\partial r}$ is ambiguous: In the limit as $\sqrt{c}\to\frac{1-r}{\tau_\theta+\tau_y}$, we have

$$\operatorname{sgn}\left\{\frac{\partial\delta_3}{\partial r}\right\}\to\operatorname{sgn}\left\{-(1-r)^3\right\}<0$$

As $r\to-\infty$, we get

$$\frac{\partial\delta_3}{\partial r}\propto -r(1-\delta_1)\left(\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)\right)^2>0$$

To show that $\frac{\partial}{\partial\tau_p}\delta_1^2\tau_p>0$:

$$\begin{aligned}
\frac{\partial}{\partial\tau_p}\left(\tau_p\delta_1^2\right)&=\delta_1^2+2\delta_1\tau_p\frac{\partial\delta_1}{\partial\tau_p}=\delta_1^2+2\delta_1\tau_p\frac{g_{\tau_s}f_{\tau_p}}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\\
&=\frac{1}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{\delta_1^2\left(g_\delta f_{\tau_s}\right)+\delta_1 g_{\tau_s}\left[2\tau_p f_{\tau_p}-\delta_1 f_\delta\right]\right\}\\
&=\frac{\delta_1^2}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{2\delta_1\left(-\chi\frac{\tau_\theta+\tau_y}{\left(\tau_\theta+\tau_y+\tau_s\right)^2}\left[1+r(\delta_1-1)\right]\delta_1^2\tau_p-(1-r)(1-\delta_1)\right)\right.\\
&\left.+2\delta_1\sqrt{c}\left[\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)-\chi\frac{\tau_s}{\tau_\theta+\tau_y+\tau_s}r\delta_1^2\tau_p+(1-r)\tau_s\right]\right\}\\
&=\frac{2\delta_1^4}{g_\delta f_{\tau_s}-g_{\tau_s}f_\delta}\left\{1-r+\frac{\sqrt{c}\delta_1^2\tau_p(1-\chi r)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y\right)}-\frac{\sqrt{c}\delta_1^2\tau_p(1-\chi)}{\delta_1+\sqrt{c}\left(\tau_\theta+\tau_y+\chi\delta_1^2\tau_p\right)}\right\}
\end{aligned}$$

Where the last step follows from lengthy but straightforward computation involving (70) and (33). Clearly, $1-r>0$ so it remains to show that the last two terms sum to a positive expression. This, however is immediate since the last is equivalent to the first with the strictly greater denominator and smaller numerator since

$$(1-\chi r)>(1-\chi)\iff\chi r<\chi\iff r<1$$

Finally, from the comparative statics of δ_3, we immediately get the comparative statics if $\gamma_3 = \frac{\delta_3}{1-\delta_3}$ which is an increasing transformation. $\square$

Proof of Proposition 8: Computing

$$\frac{\partial}{\partial \tau_p}\left(\tau_\theta + \tau_y + \tau_s + \tau_p \delta_1^2\right) = \frac{\partial \tau_s}{\partial \tau_p} + \delta_1^2 + 2\delta_1 \tau_p \frac{\partial \delta_1}{\partial \tau_p} = \frac{-g_\delta f_{\tau_p}}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta} + \delta_1^2 + 2\delta_1 \tau_p \frac{g_{\tau_s} f_{\tau_p}}{g_\delta f_{\tau_s} - g_{\tau_s} f_\delta}$$
$$\propto -g_\delta f_{\tau_p} + \delta_1^2 \left(g_\delta f_{\tau_s} - g_{\tau_s} f_\delta\right) + 2\delta_1 \tau_p g_{\tau_s} f_{\tau_p}$$

Plugging in and using (70) and (33) wherever apparent yields a linear equation in r given δ_1, which can be solved for the implicit equation

$$R_p(\chi) = \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi \delta_1^2 \tau_p\right)} \right)^2 = \chi \left(\frac{\tau_\theta + \tau_y + \tau_s + \delta_1^2 \tau_p}{\tau_\theta + \tau_y + \tau_s + \chi \delta_1^2 \tau_p} \right)^2$$

such that $\frac{\partial}{\partial \tau_p}\left(\tau_\theta + \tau_y + \tau_s + \tau_p \delta_1^2\right) > 0$ if $r < R_p$. To derive the properties of R_p, let us define

$$k(\chi, r) = \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi \delta_1^2 \tau_p\right)} \right)^2 - r = 0$$

To show that $R_p(\chi)$ is increasing in χ, we need to establish that $R_p'(\chi) = -\frac{k_\chi}{k_r} > 0$. By lengthy computation, it is easy to show that,

$$k_\chi \propto \left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y - \chi \delta_1^2 \tau_p\right)\right) + \frac{\partial \delta_1}{\partial \chi} 2\sqrt{c}(1-\chi)\tau_p \delta_1 \left(\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)\right)$$

Note that $\frac{\partial \delta_1}{\partial \chi} > 0$, and – if the first term is positive – we have $k_\chi > 0$. This is the case for valid parameters: Suppose towards a contradiction that it is not, i.e.

$$\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y - \chi \delta_1^2 \tau_p\right) < 0 \iff \tau_p > \frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}{\sqrt{c}\chi \delta_1^2}$$

But note that $r = \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi \delta_1^2 \tau_p\right)} \right)^2$ is increasing in τ_p (as a partial derivative), so this would imply that

$$\begin{aligned} r &= \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi \delta_1^2 \tau_p\right)} \right)^2 > \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2 \frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}{\sqrt{c}\chi \delta_1^2}\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi \delta_1^2 \frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)}{\sqrt{c}\chi \delta_1^2}\right)} \right)^2 \\ &= \chi \left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \frac{\delta_1}{\chi\sqrt{c}} + \frac{1}{\chi}\left(\tau_\theta + \tau_y\right)\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \frac{\delta_1}{\sqrt{c}} + \left(\tau_\theta + \tau_y\right)\right)} \right)^2 = \chi \left(\frac{\left(1 + \frac{1}{\chi}\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y\right)\right)}{2\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)} \right)^2 \\ &= \frac{(\chi+1)^2}{4\chi} = 1 + \frac{(\chi-1)^2}{4\chi} > 1 \end{aligned}$$

a contradiction. Hence we require that τ_p is smaller, otherwise the cutoff is trivial (i.e. greater than one). Hence, whenever we have an interior cutoff, we have $k_\chi > 0$.

It remains to show (to get $\frac{dr}{d\chi} > 0$) that $k_r < 0$. (with linear costs), which is the case, as

$$k_r = \frac{\partial}{\partial r}\left(\chi\left(\frac{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)}\right)^2 - r\right) = \chi(1-\chi)\,2\sqrt{c}\tau_p\delta_1 \frac{\left(\delta_1 + 2\sqrt{c}\left(\tau_\theta + \tau_y\right)\right)\left(\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)\right)}{\left[\delta_1 + \sqrt{c}\left(\tau_\theta + \tau_y + \chi\delta_1^2\tau_p\right)\right]^3}\frac{\partial\delta_1}{\partial r} - 1$$

$$\leq -1 < 0$$

In addition, we have that at the solution to $k = 0$, we always have $k_r < 0$, whence there exists a unique solution and therefore a cutoff $\mathrm{R}(\chi)$, such that $k \geq 0$ iff $r \leq \mathrm{R}(\chi)$, as we wanted to show. In addition, $\mathrm{R}' > 0$, and $\mathrm{R}(0) = 0$.

To see that the cutoff can be trivial, note that

$$\frac{\partial}{\partial\tau_p}\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right) \to 1 - \frac{1-\chi}{2\sqrt{c}\left(\tau_\theta + \tau_y\right)}$$

as $r \to 1 - \sqrt{c}\left(\tau_\theta + \tau_y\right)$, which is of ambiguous sign. □

B.3 Welfare

We now derive expression (40).

$$\begin{aligned}
\mathrm{W}(\alpha,\delta) &= \mathbb{E}\left[-(1-r)(a_i-\theta)^2 - r(a_i-\bar{a})^2\right] \qquad (73)\\
&= \mathbb{E}\left[-(1-r)(\alpha_1 s_i + \alpha_2 y + \alpha_3 p - \theta)^2 - r(\alpha_1 s_i + \alpha_2 y + \alpha_3 p - \bar{a})^2\right]\\
&= \mathbb{E}\Bigg[-(1-r)\left(\alpha_1(\theta+z_s) + \alpha_2\left(\theta+z_y\right) + \alpha_3\left(\frac{\delta_1+\delta_2}{1-\delta_3}\theta + \frac{\delta_2}{1-\delta_3}z_y + \frac{1}{1-\delta_3}z_p\right) - \theta\right)^2\\
&\quad -r\left(\alpha_1(\theta+z_s) + \alpha_2\left(\theta+z_y\right) + \alpha_3\left(\frac{\delta_1+\delta_2}{1-\delta_3}\theta + \frac{\delta_2}{1-\delta_3}z_y + \frac{1}{1-\delta_3}z_p\right) - \left(\frac{\delta_1+\delta_2}{1-\delta_3}\theta + \frac{\delta_2}{1-\delta_3}z_y + \frac{\delta_3}{1-\delta_3}z_p\right)\right)^2\Bigg]\\
&= -\frac{1}{(1-\delta_3)^2}\Bigg\{\frac{(\alpha_2(1-\delta_3)+\alpha_3\delta_2)^2 + \delta_2(\delta_2 - 2\alpha_3\delta_2 - 2\alpha_2(1-\delta_3))r}{\tau_y} + \frac{\alpha_3^2 - 2\alpha_3\delta_3 r + \delta_3^2 r}{\tau_p} + \frac{\alpha_1^2(1-\delta_3)^2}{\tau_s}\\
&\quad + \frac{(1-(1-\delta_3)(\alpha_1+\alpha_2) - \alpha_3(\delta_1+\delta_2) - \delta_3)^2 - (1 + (\delta_1+\delta_2)(1-2\alpha_3) - 2(\alpha_1+\alpha_2)(1-\delta_3) - \delta_3)(1-\delta_1-\delta_2-\delta_3)r}{\tau_\theta}\Bigg\}
\end{aligned}$$

where the last step follows after lengthy but straightforward computation. Imposing $\alpha_i = \delta_i$, we get

$$\mathrm{W}(\boldsymbol{\delta}) = -\frac{(1-r)}{(1-\delta_3)^2}\left\{\frac{\delta_2^2}{\tau_y} + \frac{\delta_3^2}{\tau_p} + \frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right\} - \frac{\delta_1^2}{\tau_s} \qquad (74)$$

Proof of Proposition 9: Taking FOC in (39), we obtain

$$\begin{aligned}
\mathrm{W}_{\delta_1} &= 2\frac{(1-r)}{(1-\delta_3)^2}\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta} - 2\frac{\delta_1}{\tau_s} = 0\\
\mathrm{W}_{\delta_2} &= -\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{\delta_2}{\tau_y} - 2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\} = 0\\
\mathrm{W}_{\delta_3} &= -\frac{2(1-r)}{(1-\delta_3)^3}\left\{\frac{\delta_2^2}{\tau_y} + \frac{\delta_3}{\tau_p} - \frac{(1-\delta_1-\delta_2-\delta_3)(\delta_1+\delta_2)}{\tau_\theta}\right\} = 0 \qquad (75)
\end{aligned}$$

Note that the last two equations simplify to a linear system in δ_2, δ_3, which we solve for them as a function of δ_1: From the first we get

$$\delta_2 = \frac{\tau_y (1 - \delta_1 - \delta_3)}{\tau_\theta + \tau_y}$$

which we plug into the second to obtain

$$\frac{\delta_3}{\tau_p} - \frac{1 - \delta_1 - \delta_3}{\tau_\theta + \tau_y} \delta_1 = 0$$

and hence

$$\delta_2 = \frac{\tau_y (1 - \delta_1)}{\tau_\theta + \tau_y + \tau_p \delta_1}$$
$$\delta_3 = \frac{\delta_1 (1 - \delta_1) \tau_p}{\tau_\theta + \tau_y + \tau_p \delta_1}.$$

Using $\frac{\delta_2}{\tau_y} = \frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}$ to simplify the W_{δ_1} condition and plugging these two expressions in, we get

$$(1-r)\,\tau_s \frac{\delta_2}{\tau_y} - \delta_1 (1 - \delta_3)^2 = 0$$
$$\delta_1 \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right) \left(\frac{\tau_\theta + \tau_y + \tau_p \delta_1^2}{\tau_\theta + \tau_y + \tau_p \delta_1} \right) - (1-r)\,\tau_s (1 - \delta_1) = 0$$

and using the envelope condition we arrive at the defining equation for the efficient outcome

$$f^\star(\delta_1) = \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right) \left(\frac{\tau_\theta + \tau_y + \tau_p \delta_1^2}{\tau_\theta + \tau_y + \tau_p \delta_1} \right) - (1-r)\,\frac{1}{\sqrt{c}} (1 - \delta_1) = 0$$

Lemma 4. *We have $f^\star_{\delta_1} > 0$ for all $\delta_1 \in (0,1)$.*

Proof of Lemma:

$$\begin{aligned}
f^\star_\delta &= \frac{\left[2\left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)\delta_1 \tau_p\right]\left(\tau_\theta + \tau_y + \tau_p \delta_1\right) - \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)^2}{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2} + (1-r)\,\frac{1}{\sqrt{c}} \\
&= \frac{\tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2} \left[\frac{(1-r)}{\sqrt{c}} \frac{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2}{\tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)} - [2\delta_1 - 1]\left(\tau_\theta + \tau_y\right) + \delta_1^2 \tau_p \right] \\
&\geq \frac{\tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)}{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2} \left[\frac{(1-r)}{\sqrt{c}} \left(\frac{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2}{\tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)} - [2\delta_1 - 1] \right) + \delta_1^2 \tau_p \right]
\end{aligned}$$

using that $\frac{1-r}{\tau_\theta + \tau_y} > \sqrt{c}$. Then, from

$$\begin{aligned}
\frac{\left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2}{\tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right)} - [2\delta_1 - 1] &\propto \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right) + \left(\tau_\theta + \tau_y + \tau_p \delta_1\right)^2 - 2\delta_1 \tau_p \left(\tau_\theta + \tau_y + \delta_1^2 \tau_p\right) \\
&= \tau_p \left(\tau_\theta + \tau_y\right) + \left(\tau_\theta + \tau_y\right)^2 + (2 - 2\delta_1)\,\tau_p \delta_1^2 \tau_p > 0
\end{aligned}$$

we obtain the result. ▲

By the Lemma above, there is a unique interior solution, as $f^\star(0) = \left(\tau_\theta + \tau_y\right) - (1-r)\frac{1}{\sqrt{c}} < 0$ by (28) and $f^\star(1) = \tau_\theta + \tau_y + \tau_p > 0$. Reformulating $f^\star(\delta_1) = 0$, we get the desired representation (42). □

Proof of Proposition 10: Comparing (33) and the final equation in (41), (43) follows immediately. □

Proof of Proposition 12: Since $f^\star_{\delta_1} > 0$ (Lemma 4), we know that $f^\star(\delta_1) < 0$ implies that there is underacquisition and $f^\star(\delta_1) > 0$ implies that there is overacquisition. Plugging the equilibrium δ_1 and using the fact that $f(\delta_1) = 0$

$$\begin{aligned} f^\star(\delta_1) &= \left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)\left(\frac{\tau_\theta + \tau_y + \delta_1^2\tau_p}{\tau_\theta + \tau_y + \delta_1\tau_p}\right) - (1-r)\frac{1}{\sqrt{c}}(1-\delta_1) \\ &= \left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)\left(\frac{\tau_\theta + \tau_y + \delta_1^2\tau_p}{\tau_\theta + \tau_y + \delta_1\tau_p} - 1\right) + \chi\frac{1}{\sqrt{c}}\frac{1}{\tau_\theta + \frac{\delta_1}{\sqrt{c}} + \tau_y}\left[1 + r(\delta_1 - 1)\right]\delta_1^2\tau_p \end{aligned}$$

Note that at $\chi = 0$, this expression is negative and hence, δ_1 is inefficiently low. As δ_1 is increasing in χ, we are below the efficient initially, but may exceed it for χ sufficiently large. There exists a χ with $\delta_1^\chi = \delta_1^\star$ iff $f^\star(\delta_1^{FC}) > 0$ (by $f^\star_\delta > 0$). We get

$$f^\star(\delta_1^{FC}) = \frac{\delta_1\tau_p}{\tau_\theta + \tau_y + \delta_1\tau_p}\left\{2\left(\tau_\theta + \tau_y\right)\delta_1 + \delta_1^3\tau_p - \left(\tau_\theta + \tau_y\right)\right\}$$

This is larger than zero iff

$$\tau_p \geq \frac{\left(\tau_\theta + \tau_y\right) - 2\left(\tau_\theta + \tau_y\right)\delta_1}{\delta_1^3}$$

the cutoff given in the proposition. In particular, if we have $\delta_1^{FC} \geq \frac{1}{2}$, the fully cursed agents always overacquires. This gives the final sufficient condition

$$\delta_1^{FC} = 1 - \frac{\sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r} \geq \frac{1}{2}$$

establishing the Proposition. □

Proof of Proposition 13. Plugging the equilibrium expressions (13),(14) for δ_2, δ_3 into W_{δ_1} and using $f(\delta) = 0$ yields

$$W_{\delta_1} = 2\frac{(1-\chi)\delta_1^3\tau_p\left(\tau_\theta + \tau_y + \tau_s\right)}{(1-r)\tau_s^2\left(\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p\right)}$$

which is zero for $\chi = 1$.

Plugging the equilibrium expressions for δ_2, δ_3 into W_{δ_2} yields

$$\begin{aligned} W_{\delta_2} =& 2\frac{\left[(1-r)\left(\tau_\theta + \tau_y + \tau_s + (1-\chi)\delta_1\tau_p\right) + (1-r\chi)\delta_1^2\tau_p\right]}{(1-r)\tau_\theta\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)^2} \\ &\cdot\left[(1-r)\chi\delta_1^2\tau_p\tau_s + (1-r)\tau_s\left(\tau_\theta + \tau_y + \tau_s\right) + \delta_1\left(\tau_\theta + \tau_y + \tau_s\right)\left(\tau_\theta + \tau_y + (1-r)\tau_s\right) + \delta_1^3\tau_p\left(\tau_\theta + \tau_y + (1-r\chi)\tau_s\right)\right] \end{aligned}$$

note that the final factor can be written as

$$\left(\tau_\theta + \tau_y + \tau_s\right)\left[\chi\frac{\tau_s}{\left(\tau_\theta + \tau_y + \tau_s\right)}\delta_1^2\tau_p(1 - r + \delta_1 r) + \delta_1\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right) + (1-r)(1-\delta_1)\tau_s\right]$$

where we recognize the factor as $f(\delta_1) = 0$, whence $W_{\delta_2}(\delta^{EQ}) = 0$.

Plugging into W_{δ_3} and simplifying with heavy use of $f(\delta) = 0$, we get

$$W_{\delta_3} = -\frac{2\chi\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)\left[\delta_1\left(\tau_\theta + \tau_y\right) - (1-r)\tau_s\right]^2}{(1-r)^2\tau_s^3\left(\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p\right)}$$

which clearly is zero in the rational case. □

Proof of Proposition 14: To determine the impact of cursedness, we compute

$$\frac{dW^{EQ}}{d\chi} = \frac{\partial W^{EQ}}{\partial\delta_1}\frac{d\delta_1}{d\chi} + \frac{\partial W^{EQ}}{\partial\delta_2}\frac{d\delta_2}{d\chi} + \frac{\partial W^{EQ}}{\partial\delta_3}\frac{d\delta_3}{d\chi}$$
$$\frac{dW^{EQ}}{d\chi}|_{\chi=0} = \frac{\partial W^{EQ}}{\partial\delta_1}|_{\chi=0}\frac{d\delta_1}{d\chi}|_{\chi=0}$$

by Proposition 13. Therefore, we have

$$\frac{dW^{EQ}}{d\chi}|_{\chi=0} = \left\{\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\} - 2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}$$

and applying the rational δ_2, δ_3, $f(\delta_1) = 0$ and the envelope condition gives

$$\begin{aligned}\frac{dW^{EQ}}{d\chi}|_{\chi=0} =& \left\{\frac{(1-r)}{(1-\delta_3)^2}\left\{2\frac{(1-\delta_1-\delta_2-\delta_3)}{\tau_\theta}\right\} - 2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}\\
=&\left\{\frac{\left((1-r)\tau_s + \delta_1^2\tau_p\right)^2}{(1-r)\tau_s^2}\left\{2\frac{\frac{(1-r)\tau_s - \tau_y\delta_1 - \delta_1\left((1-r)\tau_s + \delta_1^2\tau_p\right)}{(1-r)\tau_s + \delta_1^2\tau_p}}{\tau_\theta}\right\} - 2\sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0}\\
=&2\left\{\frac{\left((1-r)\tau_s + \delta_1^2\tau_p\right)}{(1-r)\tau_s^2}\delta_1 - \sqrt{c}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0} = 2\left\{\frac{\delta_1\tau_p c}{(1-r)}\right\}\frac{d\delta_1}{d\chi}|_{\chi=0} > 0\end{aligned}$$

which completes the proof for the derivative at $\chi = 0$.

In the fully cursed case, using Proposition 13 we know that

$$\frac{dW^{EQ}}{d\chi}|_{\chi=1} = \frac{\partial W^{EQ}}{\partial\delta_3}|_{\chi=1}\frac{d\delta_3}{d\chi}|_{\chi=1}$$

Plugging in the fully cursed weights δ_1^{FC}, δ_2^{FC}, and δ_3^{FC} into (75) yields $W_{\delta_3} = 2\sqrt{c}\delta_1^{FC}$. Therefore, we have

$$\frac{dW^{EQ}}{d\chi}|_{\chi=1} = \sqrt{c}\delta_1^{FC}\frac{d\delta_3}{d\chi}|_{\chi=1} < 0$$

where $\frac{d\delta_3}{d\chi}|_{\chi=1} < 0$ follows from

$$\frac{d\delta_3}{d\chi}|_{\chi=1} = \left(\frac{\partial\delta_3}{\partial\delta_1}\frac{\partial\delta_1}{\partial\chi}\right)|_{\chi=1} + \frac{\partial\delta_3}{\partial\chi}|_{\chi=1} = 0 - \frac{\delta_1^2\tau_p\left(\tau_\theta + \tau_y + \tau_s\right)}{(1-r)\tau_s\left(\tau_\theta + \tau_y + \tau_s + \delta_1^2\tau_p\right)} < 0$$

This establishes the result about the derivative at $\chi = 1$. □

Derivation of Formula (48): Recall that we write W_χ^{EQ} for the equilibrium welfare with degree of cursedness χ. Plugging the fully cursed equilibrium into W, we obtain

$$W_1^{EQ} = -\frac{2(1-r)\sqrt{c} - c\left(\tau_\theta + \tau_y\right)}{1-r} = -\sqrt{c}(1+\delta_1)$$

In the rational case, we instead have

$$W_0^{EQ} = -\frac{2(1-r)\sqrt{c}\delta_1^R + c\left(\tau_\theta + \tau_y + \left(\delta_1^R\right)^2 \tau_p\right)}{1-r} = -\frac{(1-r)\sqrt{c}\delta_1^R + (1-r)\sqrt{c}}{1-r} = -\sqrt{c}(1+\delta_1)$$

□

by using f.

Proof of Proposition 15: At $\chi = 0$, $\chi = 1$ we have $W = -\sqrt{c}(1+\delta_1)$. Hence, $\frac{dW}{d\tau} \propto -\frac{d\delta_1}{d\tau}$ and the comparative statics wrt τ_s follow immediately from Proposition 6. For costs, note that in the rational case, direct computation yields

$$\frac{\partial W_0^{EQ}}{\partial c} = -\frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right) + \sqrt{c}\delta_1 \tau_p}{\sqrt{c}(1-r) + 2c\delta_1\tau_p} < 0$$

which is negative by the parameter condition (28). In the fully cursed case, note that

$$\frac{\partial W_1^{EQ}}{\partial c} = -\frac{1}{\sqrt{c}}\left(\frac{(1-r) - \sqrt{c}\left(\tau_\theta + \tau_y\right)}{1-r}\right) = -\frac{\delta_1}{\sqrt{c}} < 0$$

For $\tau_p = 0$, the rational and (partially) cursed equilibria coincide, hence the above comparative statics prevail, and by continuity, this extends to small but interior τ_p.

To see the paradoxical welfare results consider welfare in the transparent limit. The welfare formula follows immediately by plugging δ^∞ from Proposition 17 into welfare, after taking $\tau_p \to \infty$. It has the following comparative statics

Lemma 5. *The welfare in the transparent limit is*

- Decreasing in τ_θ, τ_y if and only if $r > 0$ and $\chi \leq \frac{2r-1}{r}$.
- Decreasing in cursedness $\frac{\partial W^\infty}{\partial \chi} < 0$
- Decreasing in costs, unless $r < 0$, when there exists a region, $\chi \in \left(\sqrt{c}\frac{\tau_\theta+\tau_y}{1-r}, \sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{1-r\left(2-r+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}\right)$ such that higher costs increase welfare.

Proof of Lemma: To see the comparative static, note that the coefficient of $\tau_\theta + \tau_y$ is $1-2r+r\chi$. Consider the case where $r < 0$, then this expression is negative only for $\chi > 2$, so this case is irrelevant. Instead, with $r > 0$, we get that the impact of τ_θ, τ_y is negative iff $\chi \leq 2 - \frac{1}{r}$.

Furthermore

$$\frac{\partial W^\infty}{\partial \chi} = 2\sqrt{c}\frac{\sqrt{c}r\left(\tau_\theta + \tau_y\right) - (1-r)}{(1-\chi r)^2} \leq 2\sqrt{c}\frac{r(1-r) - (1-r)}{(1-\chi r)^2} = -2\sqrt{c}\frac{(1-r)^2}{(1-\chi r)^2} < 0$$

The derivative wrt. costs is

$$\frac{\partial W^\infty}{\partial c} \propto -\frac{\chi}{\sqrt{c}}(1-r)^2 + (1-2r+r\chi)\left(\tau_\theta + \tau_y\right)$$

and

$$-\frac{\chi}{\sqrt{c}}(1-r)^2+(1-2r+r\chi)\left(\tau_\theta+\tau_y\right)\geq 0$$

$$\chi\left[(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)\right]\leq\sqrt{c}(1-2r)\left(\tau_\theta+\tau_y\right)$$

Hence, there is two cases we need to consider. First, if $(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)<0$: Note that this can only be the case if $r>\frac{1}{2}$, since otherwise by (28)

$$(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)\geq(1-r)^2-r(1-r)=(1-2r)(1-r)>0.$$

Then, we obtain a lower bound for χ:

$$\chi\geq\sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)}$$

but this bound rules out increasingness in costs, since

$$\sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)}\geq 1\iff\sqrt{c}\left(\tau_\theta+\tau_y\right)\leq(1-r)$$

which is guaranteed by (28).

Second, if $(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)>0$: we get an upper bound

$$\chi\leq\sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)}.$$

The resulting interval is nontrivial only if $r<0$, as by (28)

$$\sqrt{c}\frac{\tau_\theta+\tau_y}{1-r}<\sqrt{c}(1-2r)\frac{\tau_\theta+\tau_y}{(1-r)^2-r\sqrt{c}\left(\tau_\theta+\tau_y\right)}$$

$$r\left[\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right]>r\iff r<0.$$

▲

The Lemma establishes the paradox cases. By continuity, they hold for sufficiently large τ_p.

It remains to be shown that welfare is always increasing in τ_p. Note that

$$\frac{\mathrm{d}W}{\mathrm{d}\tau_p}=\frac{\partial W}{\partial\tau_p}+\frac{\partial W}{\partial\delta_1}\frac{\mathrm{d}\delta_1}{\mathrm{d}\tau_p}+\frac{\partial W}{\partial\delta_2}\frac{\mathrm{d}\delta_2}{\mathrm{d}\tau_p}+\frac{\partial W}{\partial\delta_3}\frac{\mathrm{d}\delta_3}{\mathrm{d}\tau_p}+\frac{\partial W}{\partial\tau_s}\frac{\mathrm{d}\tau_s}{\mathrm{d}\tau_p}=\frac{\partial W}{\partial\tau_p}+\frac{\partial W}{\partial\delta_1}\frac{d\delta_1}{d\tau_p}+\frac{\partial W}{\partial\delta_3}\frac{d\delta_3}{d\tau_p}$$

Since we proved that $\frac{\partial W}{\partial\delta_2}=\frac{\partial W}{\partial\tau_s}=0$ for every χ. Simplifying this expression and plugging in for $\frac{d\delta_1}{d\tau_p}$, we obtain an expression that, after removing clearly signed factors, is proportional to a sum of positive addenda plus

$$c\chi\delta_1^4\tau_p^2\left(1+\chi+\chi^2r-3\chi r\right)$$

Now, it is clear that we need $\left(1+\chi+\chi^2r-3\chi r\right)>0$, since this term is the dominant term in τ_p and hence we otherwise have a negative expression as $\tau_p\to\infty$. But this turns out to be a nice fact of life: If $\chi\in[0,1]$, $r\leq 1$, then $h(\chi,r):=1+\chi+\chi^2r-3\chi r\geq 0$ (strictly in the interior). To see that, notice that if $r<0$, then

$$\frac{\partial}{\partial\chi}h(\chi,r)=1+2\chi r-3r=1+r(2\chi-3)>0$$

and $h(0,r) = 1 > 0$. On the contrary, if $r \in (0,1)$, then

$$1 + \chi + \chi^2 r - 3\chi r > r\left(1 + \chi + \chi^2\right) - 3\chi r = r\left(1 - 2\chi + \chi^2\right) = r(1-\chi)^2 > 0,$$

completing the proof. $\square$

C Formal Results for Section 7 (Shrewd Agent)

Proposition 18. *The action rule of the shrewd agent is*

$$\alpha_1^R = \frac{(1-(1-\delta_1)r)\tau_s^R}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \tag{76}$$

$$\alpha_2^R = \frac{(1-(1-\delta_1)r)\tau_y + \delta_2\left(r\left(\tau_\theta + \tau_y + \tau_s^R\right) - (1-r)\delta_1\tau_p\right)}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \tag{77}$$

$$\alpha_3^R = \frac{\delta_1(1-(1-\delta_1)r)\tau_p + \delta_3\left(r\left(\tau_\theta + \tau_y + \tau_s^R\right) - (1-r)\delta_1\tau_p\right)}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} \tag{78}$$

where τ_s^R *solves*

$$\tau_s^R = \frac{\alpha_1^R}{\sqrt{c}} \tag{79}$$

with complementary slackness ensuring $\tau_s^R \geq 0$ *when required.*

Proof of Proposition 18: This follows immediately from shrewd observation of the derivation of the matching coefficients and g equations. More directly, the welfare of an agent playing loadings $\boldsymbol{\alpha}$ and private precision τ_s^R while the rest of (the average of) others play $\boldsymbol{\delta}$ is derived in (73). By setting

$$\nabla_{\boldsymbol{\alpha}} W(\boldsymbol{\alpha}, \boldsymbol{\delta}) = 0$$

we find the best response coefficients as a function of others' loadings. We get

$$\begin{aligned}
0 =& (1 - \alpha_2 - \alpha_3(\delta_1+\delta_2) - \alpha_1(1-\delta_3) - \delta_3 - r(1-\delta_1-\delta_2) + \delta_3(\alpha_2 + r))\tau_s - \alpha_1(1-\delta_3)\tau_\theta \\
0 =& (1 - \alpha_2 - \alpha_3(\delta_1+\delta_2) - \alpha_1(1-\delta_3) - \delta_3 - r(1-\delta_1-\delta_2) + \delta_3(\alpha_2 + r))\tau_y - (\alpha_2(1-\delta_3) + \delta_2(\alpha_3 - r))\tau_\theta \\
0 =& \tau_y\left[-(\delta_1+\delta_2)(1-\alpha_1-\alpha_2-\alpha_3(\delta_1+\delta_2) - (1-\alpha_1-\alpha_2)\delta_3 - r(1-\delta_1-\delta_2-\delta_3))\tau_p + (\alpha_3 - \delta_3 r)\tau_\theta\right] \\
&+ 2\tau_p\tau_\theta(\delta_2(\alpha_2(1-\delta_3) + \delta_2(\alpha_3 - r)))
\end{aligned}$$

The solution to this linear system is the $\boldsymbol{\alpha}$ in the proposition, which we can plug back in welfare to obtain the expression

$$\overline{W}(\boldsymbol{\delta}) = \frac{(1-r)r}{(1-\delta_3)^2}\left[\frac{\delta_2^2}{\tau_y} + \frac{\delta_3^2}{\tau_p} + \frac{(1-\delta_1-\delta_2-\delta_3)^2}{\tau_\theta}\right] - \frac{(1-(1-\delta_1)r)^2}{\tau_\theta + \tau_y + \tau_s^R + \delta_1^2\tau_p} - c\tau_s^R$$

Differentiating this equation with respect to τ_s^R, we obtain the final condition. $\square$

Proposition 19. *The information acquired by the shrewd agent,* τ_s^R*, has the following properties.*

1. *It is bounded by the precision acquired by the cursed crowd* $\tau_s^R \leq \tau_s$*. It grows without bounds as costs vanish but can be zero even when* $\tau_s > 0$*.*
2. *If* τ_p *is sufficiently large, then it is*

- *nonmonotonic in prior and public precision,* τ_θ *and* τ_y*, possibly with an interior* activity *region; and*
- *nonmonotonic in costs* c*, possibly with an interior* inactivity *region.*

3. *It has ambiguous comparative statics w.r.t.* χ *as*

$$\frac{d\tau_s^{\mathrm{R}}}{d\chi} \propto r - 2\sqrt{c}\delta_1\tau_p$$

which is not uniformly signed.

Proof of Proposition 19: To derive equation (51) in the text, note that

$$\frac{(1-(1-\delta_1)r)^2}{\left(\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\delta_1^2\tau_p\right)^2} - c = 0$$

$$(1-(1-\delta_1)r)^2 = c\left[\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\delta_1^2\tau_p\right]^2$$

using (70) on the LHS we get after rearranging

$$\left[\frac{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}\right] = \left[\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\delta_1^2\tau_p\right]$$

From there, $\tau_s \geq \tau_s^{\mathrm{R}}$ is immediate, since

$$\frac{\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\delta_1^2\tau_p} = 1 + \frac{\chi\delta_1^2\tau_p}{\tau_\theta+\tau_y+\tau_s} \geq 1$$

For the fully cursed equilibrium, the above implies that $\tau_\theta+\tau_y+\tau_s = \tau_\theta+\tau_y+\tau_s^{\mathrm{R}}+\delta_1^2\tau_p$ and hence we have

$$\tau_s^{\mathrm{R}} = \tau_s - \delta_1^2\tau_p = \frac{1}{\sqrt{c}}\frac{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r} - \tau_p\left(\frac{1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)}{1-r}\right)^2$$

Solving for $\tau_s^{\mathrm{R}} \geq 0$, we get positive information acquisition iff $\tau_\theta+\tau_y \in \left(\frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}}, \frac{1-r}{\sqrt{c}}\right)$.

Now, for the Proposition, we have established $\tau_s^{\mathrm{R}} \leq \tau_s$. For the limit result, note that

$$\tau_s^{\mathrm{R}} = \frac{\left(\tau_\theta+\tau_y+\tau_s\right)\left(\tau_\theta+\tau_y+\tau_s+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p} - \left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right) = \frac{\left(\tau_\theta+\tau_y+\tau_s\right)\tau_s - \chi\delta_1^2\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p}$$

and that $\tau_s \to \infty$ as $c \to 0$. From there, we have

$$\tau_s^{\mathrm{R}} = \frac{\left(\tau_\theta+\tau_y+\tau_s\right)\tau_s - \chi\delta_1^2\tau_p\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)}{\tau_\theta+\tau_y+\tau_s+\chi\delta_1^2\tau_p} \to \frac{\tau_s^2}{\tau_s} \to \infty$$

We have $\tau_s^{\mathrm{R}} = 0$ for $\tau_\theta+\tau_y \leq \frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}}$ in the fully cursed case, where $\tau_s > 0$.

To see nonmonotonicity in $\tau_\theta+\tau_y$ in the general case, note that as we approach the limit (28), we have

$$\frac{d\tau_s^{\mathrm{R}}}{d\tau_y}\Big|_{\sqrt{c}=\frac{1-r}{\tau_\theta+\tau_y}} = \frac{\tau_\theta+\tau_y}{\sqrt{c}\left(2\tau_\theta+\tau_y\right)}\frac{d\delta_1}{d\tau_y} \propto \frac{d\delta_1}{d\tau_y} < 0$$

and hence, local to this value, we always get a positive τ_s^R. However, for $\tau_\theta + \tau_y \leq \chi \frac{1-r}{\sqrt{c}}$ interior, we get that for $\tau_p \to \infty$

$$\tau_s^R = \frac{\left(\tau_\theta + \tau_y + \tau_s\right)\tau_s - \chi\delta_1^2\tau_p\left(\tau_\theta + \tau_y + \delta_1^2\tau_p\right)}{\tau_\theta + \tau_y + \tau_s + \chi\delta_1^2\tau_p} \to -\delta_1^2\tau_p < 0$$

which establishes the result. Interior nonmonotonicity follows by continuity.

To see nonmonotonicity in c, note that

1. $\frac{d\tau_s^R}{dc} < 0$ in the limit (28) ($\sqrt{c} = \frac{1-r}{\tau_\theta+\tau_y}$). To see this, take the representation above, plug 33, take the derivative, set $\sqrt{c} = \frac{1-r}{\tau_\theta+\tau_y}$, $\delta_1 = 0$, then we get

$$\frac{d\tau_s^R}{dc}\Big|_{\sqrt{c}=\frac{1-r}{\tau_\theta+\tau_y}} \to \frac{2(1-r)^4\frac{\partial\delta_1}{\partial c}}{\left(\tau_\theta + \tau_y\right)^2} \propto \frac{\partial\delta_1}{\partial c} < 0$$

2. $\tau_s^R = 0$ at the limit (28), as $0 \leq \tau_s^R \leq \tau_s = 0$. Therefore, by (1.) and (2.), $\tau_s^R > 0$ local to this upper bound on costs.
3. $\tau_s^R \to \infty$ as $c \to 0$, as shown above.
4. For any c satisfying the existence of a transparent limit equilibrium ($\sqrt{c} \leq \chi\frac{1-r}{\tau_\theta+\tau_y}$), there exists a τ_p sufficiently large such that $\left(\tau_s^R\right)^{\text{FOC}} < 0$. To see this, pick an interior c. Then, $\delta_1^\infty > 0$ and as $\tau_p \to \infty$

$$\left(\tau_s^R\right)^{\text{FOC}} \to -\delta_1^2\tau_p < 0$$

This establishes nonmonotonicity, as τ_s^R is large for small c, zero for intermediate c, but nonzero local to $\sqrt{c} = \frac{1-r}{\tau_\theta+\tau_y}$.

To analyze the derivative in χ, we compute

$$\frac{d\tau_s^R}{d\chi} \propto r - 2\sqrt{c}\delta_1\tau_p$$

It is apparent that τ_s^R is decreasing for $r \leq 0$ and that it is increasing as $r \to 1 - \sqrt{c}\left(\tau_\theta + \tau_y\right)$ when this is positive, as then $\delta_1 \to 0$. To see that we can have a hump shape, note that δ_1 is increasing in χ and hence

$$\frac{d^2\tau_s^R}{d\chi d\chi} \propto -\frac{d\delta_1}{d\chi} \leq 0$$

around $\frac{d\tau_s^R}{d\chi} = 0$, which establishes a hump-shape (but, importantly, not necessarily concavity!). Clearly, all these comparative statics only apply for interior solutions, otherwise $\tau_s^R \equiv 0$ locally. □

Remark (The Shrewd Agent in the Transparent Limit). Consider the limit as $\tau_p \to \infty$. Since $\delta_1^\infty > 0$ whenever a limit equilibrium exists, the rational agent can exactly infer the state. Therefore, he does not acquire or use private information[35] and relies solely on the aggregative signal for information

$$\alpha_1^R \to 0, \quad \tau_s^R \to 0, \quad \alpha_2^R \to \frac{-(1-r)\sqrt{c}\tau_y}{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)} < 0, \quad \alpha_3^R \to \frac{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)r}{1 - r - \sqrt{c}\left(\tau_\theta + \tau_y\right)} > 0 \tag{80}$$

The apparent anti-imitation in $\alpha_2^R < 0$ allows the shrewd agent to filter out the over-reliance of the cursed crowd on the public signal.

Proposition 20. *Suppose parameters are such that $\tau_s^R > 0$.*

[35] Indeed, notice that the interval (53) vanishes as $\tau_p \to \infty$.

- *If* $r \leq 0$, *then* $\left.\frac{dW_\chi^R}{d\chi}\right|_{\chi=1} \geq 0$. *However,*
- *for* r *sufficiently large,* $\left.\frac{dW_\chi^R}{d\chi}\right|_{\chi=1} \leq 0$.

Proof of Proposition 20. For χ, we plug the general δ into the welfare expression, take derivative wrt χ, set $\chi = 1$, $\delta_1 = \delta_1^{FC}$ and we get

$$\begin{aligned}\frac{\mathrm{d}W_\chi^R}{\mathrm{d}\chi} &= -\frac{2\sqrt{c}(1-r)\tau_p\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2\left[r\left(\tau_\theta+\tau_y+\tau_s^R+\tau_p\right)-\tau_p\left(1-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\right]\left(1-r\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\right)^2}{[\cdot]^2\left[1+\sqrt{c}\tau_p\left(1-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2+r^2\left(1+\sqrt{c}\left(\tau_\theta+\tau_y+\tau_p\right)\right)-r\left(2-2c\tau_p\left(\tau_\theta+\tau_y\right)+\sqrt{c}\left(\tau_\theta+\tau_y+2\tau_p\right)\right)\right]} \\ &\propto -\frac{2\sqrt{c}\tau_p\delta_1^2(1-r)^3\left[r\left(\tau_\theta+\tau_y+\tau_s^R+\tau_p\delta_1\right)-\tau_p\delta_1\right]}{1+\sqrt{c}\tau_p\left(1-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2+r^2\left(1+\sqrt{c}\left(\tau_\theta+\tau_y+\tau_p\right)\right)-r\left(2-2c\tau_p\left(\tau_\theta+\tau_y\right)+\sqrt{c}\left(\tau_\theta+\tau_y+2\tau_p\right)\right)}\end{aligned} \tag{81}$$

Consider first the case of $r > 0$, which implies that $1-\sqrt{c}\left(\tau_\theta+\tau_y\right) > 0$. As $r \to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$, we have for the above expression

$$\to -2\tau_p\left(\tau_\theta+\tau_y\right)\sqrt{c}\left(1-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)\delta_1^2 \leq 0$$

This converges to zero from below since $\delta_1 \to 0$ as $r \to 1-\sqrt{c}\left(\tau_\theta+\tau_y\right)$. Therefore, for large χ, the shrewd agent prefers a less cursed environment if r is big.

Consider now $r \leq 0$. Note that the numerator in (81) is always negative. Furthermore, note that the denominator $v(r)$ is positive for $r = 0$. We will show that it is positive for all $r \leq 0$ and hence the expression is positive for all $r \leq 0$. First, note that v is a convex quadratic function in r. Minimizing, we find that the minimizer and minimum, resp., are given by

$$\begin{aligned}r^* &\propto 2-2c\tau_p\left(\tau_\theta+\tau_y\right)+\sqrt{c}\left(\tau_\theta+\tau_y+2\tau_p\right) \\ v^* &\propto -1+4c\tau_p\left(\tau_\theta+\tau_y\right)\end{aligned}$$

If $v^* > 0$, we are done. If it is negative, it is easy see that r^* must be positive. But then, the fact that $v(0) > 0$ implies that $v(r) > 0$ for all $r \leq 0$. □

Proposition 21. *The welfare of a shrewd agent in a fully cursed population,* W_1^R, *has the following properties.*

1. *It is strictly increasing in* τ_p.
2. *It has ambiguous comparative statics with respect to* τ_y. *In particular,*
   - *At the boundary of the activity region, (i.e.* $\tau_y = \frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}} - \tau_\theta$*), we have* $\frac{\mathrm{d}W_1^R}{\mathrm{d}\tau_y} < 0$,
   - *for* τ_y *large (i.e. local to the nontriviality limit* $\tau_y = \frac{(1-r)}{\sqrt{c}} - \tau_\theta$*), we have* $\frac{\mathrm{d}W_1^R}{\mathrm{d}\tau_y} > 0$.
3. *It is decreasing in* c *whenever* $\tau_s^R > 0$, *but it has ambiguous comparative statics with respect to* c *if* $\tau_s^R = 0$. *In particular,*
   - *If* $\tau_p \geq \frac{1}{\sqrt{c}}$ *and* r *sufficiently negative, then* $\frac{\mathrm{d}W_1^R}{\mathrm{d}c} > 0$.

Proof of Proposition 21: To obtain the welfare of the shrewd agent in the fully cursed equilibrium, we simply plug action rule (76)-(50) and the equilibrium δs into the welfare equation to obtain, for the unconstrained case

$$W_1^{R,\tau_s^R>0} = -2\sqrt{c} + \frac{-2c^{3/2}(1-r)\tau_p\left(\tau_\theta+\tau_y\right) + c^2\tau_p\left(\tau_\theta+\tau_y\right)^2 + c(1-r)\left(\tau_\theta+\tau_y+(1-r)\tau_p\right)}{(1-r)^2}$$

as well as for the constrained case, where we leave the expression in general form both for compactness and ease of analysis

$$W_1^{R,\tau_s^R=0} = -\frac{(1-r)r\delta_2^2}{\tau_y} - \frac{(1-r)r(1-\delta_1-\delta_2)^2}{\tau_\theta} - \frac{(1-(1-\delta_1)r)^2}{\tau_\theta+\tau_y+\delta_1^2\tau_p}$$

For transparency, we obtain by direct computation for an interior solution $\frac{\partial W_1^{R,\tau_s^R>0}}{\partial \tau_p} = c\frac{\left(1-r-\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{(1-r)^2} = c\delta_1^2 > 0$, and through an envelope argument from the general expression, we get form corner solutions $\frac{\partial W_1^{R,\tau_s^R=0}}{\partial \tau_p} = \frac{\delta_1^2(1-(1-\delta_1)r)^2}{\left(\tau_\theta+\tau_y+\delta_1^2\tau_p\right)^2} > 0$.

For τ_θ, τ_y: Consider the derivative of $W_1^{R,\tau_s^R>0}$ and let $\tau_\theta+\tau_y \to \frac{(1-r)\left(1-\frac{1}{\tau_p\sqrt{c}}\right)}{\sqrt{c}}$. Then, we get

$$\frac{\partial W_1^{R,\tau_s^R>0}}{\partial \tau_y} \to c\frac{1-r-(1-r)2\sqrt{c}\tau_p + 2c\tau_p\left(\tau_\theta+\tau_y\right)}{(1-r)^2} = \frac{c}{1-r}\left(1-2\sqrt{c}\tau_p\delta_1^{FC}\right) = -\frac{c}{1-r} < 0$$

Note that this result also holds for $W_1^{R,\tau_s^R=0}$, as the value function is $\mathcal{C}^1$. Taking instead the limit as $\tau_\theta+\tau_y \to \frac{(1-r)}{\sqrt{c}}$ (where we always are at an interior solution), we have

$$\frac{\partial W_1^{R,\tau_s^R>0}}{\partial \tau_y} \to \frac{c}{1-r}\left(1-2\sqrt{c}\tau_p\delta_1^{FC}\right) = \frac{c}{1-r}$$

For c, let us first demonstrate a setting where an increase in costs is beneficial for the shrewd agent. Consider constrained welfare, let $r \to -\infty$ and hence $\delta_1 = 1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)} \to 1$, $\delta_2 = \frac{\sqrt{c}}{1-r}\tau_y \to 0$. Then

$$W_1^{R,\tau_s^R=0} \to -\frac{rc\tau_y}{(1-r)} - \frac{r\tau_\theta c}{(1-r)} - \frac{\left(1-\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}r\right)^2}{\tau_\theta+\tau_y+\tau_p} \to c\tau_\theta + \tau_y c - \frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{\tau_\theta+\tau_y+\tau_p}$$

and

$$\frac{\partial}{\partial c}\left(c\tau_\theta + \tau_y c - \frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)^2}{\tau_\theta+\tau_y+\tau_p}\right) = \tau_\theta+\tau_y - \frac{\left(\tau_\theta+\tau_y\right)}{\sqrt{c}}\frac{\left(1+\sqrt{c}\left(\tau_\theta+\tau_y\right)\right)}{\tau_\theta+\tau_y+\tau_p}$$
$$= \left(\tau_\theta+\tau_y\right)\left(\frac{\tau_p - \frac{1}{\sqrt{c}}}{\tau_\theta+\tau_y+\tau_p}\right) > 0$$

since we consider the case $\tau_s^R = 0$, and therefore $\tau_p > \frac{1}{\sqrt{c}}$.

Generally, we can also have $\frac{\partial}{\partial c}W_1^R < 0$. To see this, consider the case of $r = 0$. Then

$$W^R|_{r=0} = \max_{\tau_s^R \geq 0} -\frac{1}{\tau_\theta+\tau_y+\tau_s^R+\delta_1^2\tau_p} - c\tau_s^R$$

where we may or may not have a corner solution. In either case, welfare is decreasing in c because δ_1 is decreasing in c and an envelope argument.

At an interior solution (which occurs for $\tau_p < \frac{1}{\delta_1 \sqrt{c}}$), we get

$$\begin{aligned}\frac{\partial}{\partial c} W_1^{R,\tau_s^R>0} &= -\frac{1}{\sqrt{c}} + \frac{-3\sqrt{c}(1-r)\tau_p\left(\tau_\theta+\tau_y\right) + 2c\tau_p\left(\tau_\theta+\tau_y\right)^2 + (1-r)\left(\tau_\theta+\tau_y+(1-r)\tau_p\right)}{(1-r)^2} \\ &= \frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \delta_1\tau_p\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right)\end{aligned}$$

which is always negative by condition (28) if $\delta_1 \leq \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}$. If this is violated, we have

$$\begin{aligned}\frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \delta_1\tau_p\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) &\leq \frac{\left(\tau_\theta+\tau_y\right)}{(1-r)} - \frac{1}{\sqrt{c}} + \frac{1}{\sqrt{c}}\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) \\ &= -\frac{\delta_1}{\sqrt{c}} + \frac{1}{\sqrt{c}}\left(\delta_1 - \frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) = \frac{1}{\sqrt{c}}\left(-\frac{\sqrt{c}\left(\tau_\theta+\tau_y\right)}{(1-r)}\right) < 0\end{aligned}$$

concluding the proof. □

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