THE NOISE IS IN THE MIND: EXISTENCE OF TRADING EQUILIBRIA WITH TRANSPARENT PRICES

FRANZ OSTRIZEK ELIA SARTORI

Abstract. We investigate the behavioral foundations of informed trade. We extend the canonical Kyle (1989) model to allow for a wide range of misperceptions about the information environment (e.g. overconfidence and correlation delusion) as well as the market clearing condition (e.g. underestimation of market impact) and ask when a trading equilibrium exists.

Equilibrium exists if either (i) traders perceive the pricing rule as noisy, or (ii) traders sufficiently overestimate their informational advantage or underestimate their market impact. (i) provides a cognitive reinterpretation of noise traders: what matters for existence is perceived, not actual, noise in market clearing. (ii) yields a tractable linear model but requires substantial biases; unlike noise, infinitesimal biases do not suffice.

Our existence conditions have direct implications for trade volume. When they are satisfied, informed trading is self-sustaining: it persists even as noise trading vanishes, and the share of informed volume grows without bound. When violated, informed trading is parasitic on noise trading and vanishes with it. The same biases that make informed trade possible thus also make it substantial.

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1. Introduction

Models of informed trading lie at the heart of the literature on market microstructure in economics and finance. Across a wide range of such models, there is a well understood problem of equilibrium existence that at least dates back to the classic price paradox (Grossman & Stiglitz, 1980): If all dispersed information is aggregated through the market price then individual private information is superfluous and traders should not use it; but if traders don't use private information, the market price contains no fundamental information and traders should use their own. Aggregation through prices means that private information is superfluous if (and only if) it is used, making a trading (as well as a no-trade) equilibrium impossible.

A common way to resolve this vicious cycle – and to obtain equilibrium existence – is to assume that markets are also populated by a fringe of noise traders (Black, 1986; Kyle, 1985) that inject aggregate noise to the market price. They do so by trading based on non-fundamental information, usually justified as an exogenous liquidity or hedging demand or as the result of unsophisticated behavioral traders. In the words of Black (1986) "Noise makes financial markets possible, but also makes them imperfect".

In stark cognitive opposition to the random actions of noise traders, informed traders are assumed perfect Bayesians who correctly interpret their own information as well as the information contained in prices. This assumption has been challenged in the vast literature on behavioral finance, which documents and models the range of market imperfections caused by the biases and misperceptions of market participants, such as overconfidence (Daniel et al., 1998; Odean, 1998), cursedness (Eyster, Rabin & Vayanos, 2019), or misperceptions of correlation (Banerjee, 2011).

In this paper, we ask whether biases and misperceptions make financial markets not only imperfect, but also *possible*. We consider a workhorse model of informed trade (Kyle, 1989) in a version without noise traders, but a wide range of behavioral parameters. Our objective environment is completely symmetric and prices are determined in fully revealing, frictionless manner. Our agents, however, may misperceive the information environment, thinking themselves to be more informed than others. Similarly, they may misperceive the market clearing rule, believing themselves to be more (or less) important, the price to be set or communicated with noise, or the market to be deeper than it truly is.

As is well understood, there is no equilibrium in the rational benchmark. How does the strength and interaction of these biases affect equilibrium existence? First, we show that an equilibrium exists as long as traders perceive the price to be noisy. Indeed, we can interpret the noise trader in the classic model as cognitive noise: What matters (for existence and the equilibrium) is not the real market clearing condition, but the market clearing condition perceived by the traders.[1] Second, in an economy without noise, we show that equilibrium existence requires sufficiently strong misperceptions, what we call the *self-sustaining trade condition*. Existence is ensured when traders overestimate the precision of their private information (relative to that of others reflected in the market price) and underestimate their market impact (relative to the depth of the market) to a sufficient degree.

The fact that substantial biases are required in the absence of noise traders, though an infinitesimal noise trader is sufficient, reflects a structural difference between the model with and without noise. In the former, the signal-to-noise ratio of the price is endogenous, and trading activity scales to ensure an equilibrium, no matter how small the variance of the noise. In the latter, the price is fully informative in every informative equilibrium, and a sufficiently strong bias is required to ensure that individual traders use their private information. Another consequence of this distinction is that the model without noise traders is highly tractable, as it preserves the linearity inherent in the Gaussian-CARA setting.

This structural difference has stark implications for trade volume. While arbitrarily small noise suffices for existence, the resulting equilibria entail essentially no trade, as informed trading activity must scale down to match the noise trading it feeds on. Plausible levels of informed trade therefore require substantial noise trading which, taken literally, must originate either from few "big" noise traders or from a mass of "small" traders coordinating through a factor that the informed side of the market cannot observe. Cognitive noise is immune to this large-correlated-noise critique: the noise with which market participants perceive market clearing exists only in the minds of the traders. Systematic biases provide an alternative resolution; when they satisfy the self-sustaining trade condition, informed trade persists even as noise trading—real or perceived—vanishes.

1.1. **Related Literature.** Our model is a behavioral extension of the demand function equilibrium framework developed by Kyle (1989). In that framework, equilibrium existence requires noise traders whose demand masks informed trades; we show that cognitive biases about the information environment or market clearing can play the same role. Kyle, Obizhaeva, et al. (2018) study overconfident traders in continuous time and find that sufficient disagreement about signal precision is required for equilibrium. We allow for a richer set of biases including misperceptions of correlation and market impact, not only signal precision, and provide sharp existence thresholds that characterize exactly how much bias is needed.

Our paper contributes to the literature escaping the no-trade theorem (Milgrom & Stokey, 1982; Morris, 1994), which establishes that under common priors, traders

[1] Of course, when analyzing the market, e.g. computing price volatility, it will make a difference whether the noise traders are present in the market or merely in the mind of the traders.

cannot profit from information differences alone. Heterogeneous priors can enable speculation (Harrison & Kreps, 1978; Scheinkman & Xiong, 2003). We provide a parametric characterization, within a model of strategic trading, of precisely which misperceptions about the information environment (non-common priors about the information structure) generate the interim disagreement necessary for trade.

Our cognitive noise interpretation connects to the literature on noise trader foundations. De Long et al. (1990) model noise traders with stochastic beliefs who create risk that deters arbitrage and may thereby survive; Spiegel & Subrahmanyam (1992) endogenize noise through hedging motives. Our approach differs along two dimensions. First, we reinterpret noise as cognitive: what matters for existence is traders' perception of noise in market clearing, not its actual presence. Second, we distinguish stochastic from systematic biases. Stochastic noise generates an endogenous signal-to-noise ratio that scales to support equilibrium for any noise variance, hence an infinitesimal amount suffices. Systematic biases in our linear model lack this scaling property and require sufficient magnitude to overcome the no-trade logic.

The behavioral biases we study are grounded in extensive empirical evidence. A large literature studies overconfidence (Daniel et al., 1998; Kyle & Wang, 1997; Odean, 1998), often focusing on momentum and overreaction.[2] Our results on misperception based trade volume are related to results showing that overconfidence increases trade volume; we show that it also breaks the link between liquidity-driven and information-driven volume. Barber & Odean (2000) document that individual investors trade excessively and that this correlates with survey measures of overconfidence. Odean (1998) observes that overconfidence can restore existence in a version of Diamond & Verrecchia (1981) without noise traders (see also Varian, 1989). Wang (1998) shows that overconfidence of the informed trader in Kyle (1985) is detrimental for existence; with only a single informed trader, overconfidence does not increase market liquidity (see also Remark 2 on Caballé & Sákovics (2003)). Kyle, Obizhaeva, et al. (2018) and Kyle, Obizhaeva, et al. (2023) rely on overconfidence for existence in the absence of noise traders and find a condition equivalent to our condition 3.8. We do not focus on a specific bias and its consequences, but ask what kind of bias is needed for equilibrium to exist at all and the common consequences of misperception-driven equilibria.

[2] Another literature, following Harrison & Kreps (1978) studies the impact of heterogeneous beliefs on asset prices in the presence of short-selling constraints.

2. Model and Equilibrium Definition

We aim to model traders who may suffer from a range of misperceptions of the information environment and of market clearing. Before introducing these biases, let us describe the objective environment.

2.1. **Objective Environment.** We conduct our analysis in the workhorse model of Kyle (1989). There are N ex-ante identical traders with CARA utility and risk aversion parameter ρ. They trade a single risky zero net supply asset with liquidation value $\nu \sim \mathcal{N}(0, \tau_o^{-1})$ by submitting demand schedules $x_i(p)$. Before submitting their demand, every trader receives a private signal $s_i \sim \mathcal{N}(\nu, \tau_s^{-1})$ independently across traders. The price is set by a market maker to clear the market

$$\sum_i x_i(p) = 0$$

and payoffs, $x_i(\nu - p)$, realize.

We focus on symmetric linear equilibria in which traders use a linear demand schedule, $x_i = \beta s_i - \gamma p$. Given this demand schedule, the market clearing price can be written as

$$p = p_i + \lambda x_i \tag{2.1}$$

where p_i is the intercept of the residual supply curve for trader i and λ denotes her market impact. Conditioning on the price, then, allows traders to infer about the average signal of others, and we write $\mathbb{E}[\nu|s_i, p]$ and $\mathbb{V}[\nu|s_i, p]$ for her posterior expectation (resp. variance) of the fundamental value. Note that conditioning on p or on the intercept p_i is equivalent given s_i, as the two differ only by the trader's own demand, a deterministic function of (s_i, p). The optimal demand is given by

$$x_i = \frac{\mathbb{E}\left[\nu|s_i, p\right] - p_i}{2\lambda + \rho\mathbb{V}\left[\nu|s_i, p\right]}. \tag{2.2}$$

2.2. **Misperceptions.** We assume throughout that our traders have the above common prior about the fundamental value of the asset, the utility specification, and we only consider biases that preserve the jointly Gaussian structure of the environment. Furthermore, we focus on symmetric environments, in the sense that all traders have the same perception of the information environment and the market clearing condition.

Consequently, we can restrict attention to symmetric equilibria. The best-response condition (2.2) also holds in our setting with misperceptions, though clearly the specific biases will affect the specification of the expectation $\mathbb{E}$, variance $\mathbb{V}$ and perceived market impact λ.

2.2.1. *Perceived Information Environment.* According to trader i, the signals of herself and the other traders are distributed according to

$$\left(\begin{array}{c} s_i \\ \boldsymbol{s_{-i}} \end{array}\right) \sim \mathcal{N}\left(\boldsymbol{\nu}, \tau_s^{-1}\left(\begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{array}\right)\right) \tag{2.3}$$

where $\Sigma_{11} \in \mathbb{R}_+$ captures misperception of the trader's own signal such as overconfidence, $\Sigma_{22} \in \mathbb{R}^{N-1\times N-1}$ captures misperceptions about the signals of others, such as dismissiveness and correlation delusion and $\Sigma_{12} \in \mathbb{R}^{N-1}$ captures misperceptions about the link between the signal of the trader herself and that of others, e.g. information projection. The only restriction we impose on the perceptions of the agent is that $\Sigma = \left(\begin{array}{cc} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{array}\right)$ is a valid variance-covariance matrix, i.e. that it is positive semi-definite.

For instance, overconfidence corresponds to $\Sigma_{11}^{-1} = \xi > 1$; dismissiveness and correlation delusion to $\Sigma_{22} = I\delta\left(1-\phi\right) + \mathbf{1}^T\delta\phi\mathbf{1}$, where δ captures dismissal of others' precision and $\phi > 0$ a perceived common component in the errors of others; and information projection to $\Sigma_{12} = \alpha\mathbf{1}$. We develop these examples in Section 3.1.

2.2.2. *Perceived Market Clearing.* To understand how the price is determined, the trader uses her perceived market clearing rule

$$\left(\begin{array}{cccc} \Lambda_1, & \Lambda_2, & \Lambda_z\tau_s^{-1/2}, & \Lambda_p \end{array}\right) \cdot \left(\begin{array}{c} x_i \\ \boldsymbol{x}_{-i} \\ z \\ -\gamma p \end{array}\right) = 0 \tag{2.4}$$

where the parameters $\Lambda_1 \in \mathbb{R}_+, \Lambda_2 \in \mathbb{R}_+^{N-1}$ measure the relative misperception of the impact of the demand of the traders in determining the market price; $z \sim \mathcal{N}(0,1)$ is perceived noise in market clearing, e.g. because traders believe that there is noise trader demand, with $\Lambda_z \geq 0$ denoting the perceived standard deviation of this noise relative to private information and $\Lambda_p \geq 0$ denotes additional market depth perceived to be provided by uninformed traders. For instance, a trader with $\Lambda_z > 0$ perceives the noise trader demand of the classic Kyle model (whether it is actually present or the noise is only in the mind) while the size of Λ_1 relative to Λ_2 captures a misperception of her own market impact.

This perceived market clearing condition is used by the traders both to extract information from the price as well as to evaluate their market impact. Rearranging (2.4) we obtain the following linear subjective pricing equation of the form (2.1)where

$$\lambda = \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} \tag{2.5}$$

represent how much agents' think their demand will impact the price, while

$$p_i = \frac{\beta\Lambda_2 \boldsymbol{s}_{-i} + \Lambda_z \tau_s^{-1/2} z}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} \tag{2.6}$$

denotes the subjective intercept (the price that would realize if she chooses to refrain from trading).

The economy is characterized by the set of behavioral parameters $\vartheta = (\Sigma, \Lambda)$, and an objective environment $\tau = [\tau_0, \tau_s, \rho, N]$. We refer to $\vartheta^R = \left(I_{N\times N}, \begin{pmatrix} 1 & , \mathbf{1}, & 0, & 0 \end{pmatrix}\right)$ as the rational economy and to $\vartheta^K_{\Lambda_z} = \left(I_{N\times N}, \begin{pmatrix} 1 & , \mathbf{1}, & \Lambda_z, & 0 \end{pmatrix}\right)$ as the Λ_z-Kyle economy for some $\Lambda_z \in (0, \infty)$.

We introduce a useful taxonomy of the class of economies we will consider.

Definition 1. An economy is **regular** if $\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T \neq 0$ and $\mathbf{1}\Lambda_2^T + \Lambda_p > 0$. An economy is **linear** if it is regular and $\Lambda_z = 0$.

Regularity is a mild assumption that ensures that the price (subjectively) contains some information about the fundamental even after conditioning on the private signal and a non-solipsism condition. As their analysis is qualitatively different, we study the knife-edge case of non-regular economies separately in Appendix B.1. Linearity further requires that traders do not perceive an exogenous shifter in the market clearing equation. The reason for the moniker will be apparent in the next section. For the moment, we just notice that the rational economy is linear while the Kyle economy is not linear.

Throughout the main text, we maintain the following restrictions on the economy.

Assumption 1. *The economy is regular (Definition 1), there are at least three traders, $N \geq 3$, and $\Lambda_1 < \mathbf{1}\Lambda_2^T + \Lambda_p$.*

2.2.3. *Discussion: our class of misperceptions.* The misspecified economies summarized by the parameters $\vartheta = (\Sigma, \Lambda)$ allow us to consider a broad class of misperceptions about the information as well as institutional environment. As the examples in Section 3.1 clarify, many commonly studied biases such as overconfidence, neglect of market power, and information projection fall within our setup. However, our specification is not without loss of generality: in particular, we consider economies with a representative trader that is a misspecified Bayesian agent. Misspecified Bayesian traders have equilibrium awareness, i.e. implicit in the fixed-point formulation is that traders use the equilibrium loadings (β, γ) together with their subjective beliefs about the information environment and the market clearing condition when extracting information from prices to compute their best-response. In other words, our traders know that competitors put weight β on their private signal, though they might misunderstand (in an essentially arbitrary way) the precision and covariance of the competitors' signals as well as the way their action determines the market price. Indeed, these equilibrium loadings are all that a trader needs to know about her competitors. [3]

Understanding the types of biases that lead to existence even outside the misspecified Bayesian paradigm is a natural extension of our analysis. We make a first step towards this generalization in Appendix B.2 where we study existence when traders form beliefs according to the mixture model (not representable with (2.3)-(2.4)) of cursed equilibrium, e.g. in a strategic version of the model considered by Eyster, Rabin & Vayanos (2019).

The market environment admits two readings, and we think both are suitable. Read literally, market clearing describes a uniform-price auction in which traders submit demand schedules to a centralized protocol; the natural misperceptions then concern the composition of the order flow, as when a trader perceives non-fundamental demand ($\Lambda_z > 0$) or additional depth ($\Lambda_p > 0$) in (2.4) that is not actually there. Alternatively, market clearing is a reduced form for a decentralized trading process. Under this reading, the clearing rule is itself an approximation in the mind of the trader, and misperceiving one's own impact relative to that of others (Λ_1 vs. Λ_2) is a natural simplification: ignoring one's market impact, for instance, formally is a misperception of the market clearing rule, but appears to be an often reasonable approximation for a trader navigating a decentralized market. Under either reading, we view the model as an abstract benchmark in the tradition of Kyle (1989), not as a calibrated descriptive tool to be taken to the data directly. The question it isolates is which misperceptions can, in principle, sustain informed trade in the absence of noise traders.

2.3. **Inference and Equilibrium.** To obtain $\mathbb{E}\left(\nu|s_i,p\right), \mathbb{V}\left(\nu|s_i,p\right)$ it is convenient to transform the price into a signal $\hat{p}_i$ that is (subjectively) mean ν and conditionally independent of s_i, that is

$$\begin{pmatrix} s_i \\ \hat{p}_i \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \begin{pmatrix} \hat{\tau}_s & 0 \\ 0 & \hat{\tau}_p \end{pmatrix}\right) \tag{2.7}$$

so that

$$\mathbb{E}\left(\nu|s_i,p\right) = \frac{s_i\hat{\tau}_s + \hat{p}_i\hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0}, \qquad \mathbb{V}\left(\nu|s_i,p\right) = \frac{1}{\hat{\tau}_p + \hat{\tau}_s + \tau_0}.$$

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[3]Furthermore, our traders use a single consistent (though perhaps misspecified) model of market clearing, both to assess the information contained in the price and to understand their market impact. This is in contrast to the typical price-taking assumption often employed in rational expectations equilibria formulations with finitely many traders; for a discussion of this issue, see Hellwig (1980).

Lemma 1. *The conditionally independent representation (2.7) results from letting*

$$\hat{p}_i = \frac{\frac{(\mathbf{1}\Lambda_2^T + \Lambda_p)\gamma}{\beta} p_i - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} \tag{2.8}$$

and

$$\hat{\tau}_p^{-1} = \tau_s^{-1} \frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2} \tag{2.9}$$

$$\hat{\tau}_s^{-1} = \tau_s^{-1}\Sigma_{11} \tag{2.10}$$

In words, the trader strips from the residual-supply intercept p_i the part that is driven by her own signal, which is informative about her competitors' signals whenever she perceives them to be correlated with hers. She then rescales the difference so that the result is an unbiased signal of ν that is (subjectively) conditionally independent of s_i.

We therefore arrive at the consistency condition

$$\beta s_i - \gamma p = \frac{\mathbb{E}(\nu|s_i,p) - p_i}{2\lambda + \rho\mathbb{V}(\nu|s_i,p)} = \frac{\frac{s_i\hat{\tau}_s + \hat{p}_i\hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0} - p_i}{2\lambda + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}} \tag{2.11}$$

where the variables on the right hand side are given as a functions of (β, γ) above.

We are now ready to define a trading equilibrium.

Definition 2. Given an economy (ϑ, τ) an equilibrium is a pair (β, γ) such that

i) (β, γ) satisfy the matching coefficients condition (2.11)
ii) the second order condition $2\lambda(\tau_0 + \hat{\tau}_s + \hat{\tau}_p) + \rho > 0$ is satisfied.

Essentially, an equilibrium exists if the matching coefficient operator has a fixed point that satisfies the (subjective) second-order condition. In the rational economy no equilibrium exists while the Kyle economy always has an equilibrium.

Lemma 2. *If $\vartheta = \vartheta^R$, then there is no equilibrium. For any $\Lambda_z \in (0,\infty)$, $\vartheta^K_{\Lambda_z}$ admits a unique equilibrium.*

3. Existence

The mechanics of equilibrium existence differ between the linear and the nonlinear case. In the nonlinear case, traders perceive the presence of noise traders. By Lemma (1), the precision of the residual information contained in the price is given by

$$\hat{\tau}_p(\beta) = \tau_s\left[\frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2} + \frac{\Lambda_z^2}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2}\frac{1}{\beta^2}\right]^{-1} \tag{3.1}$$

The extent the private signal is used by traders (β) determines the signal-to-noise ratio of the price. Notice that, whenever $\Lambda_z > 0$ then as $\beta \to 0$ the second term explodes and $\hat{\tau}_p(\beta)$ vanishes: no matter its variance, if traders perceive an exogenous shifter in the market clearing equation then the precision of the price goes to zero if the private signal is just barely used. This endogenous precision adjustment is key to ensure equilibrium existence: Intuitively, if the price were too informative and crowded out the use of private information, its precision would be endogenously depressed, causing traders to rely more on private information and thereby supporting equilibrium existence. This will be possible as long as the perceived residual supply curve for the asset is sufficiently flat.

In the linear case ($\Lambda_z = 0$), the perceived precision of the price is instead independent of β,

$$\hat{\tau}_p = \tau_s \frac{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2}{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}. \tag{3.2}$$

Since there is no (perceived) noise, the price fully reveals the weighted sum of signal realizations $\Lambda_2 \boldsymbol{s}_{-i}$, independently of β, as long as it is nonzero. The perceived precision of prices is the same in all such trading equilibria and depends solely on the (primitive) biases. Existence then relies on a sufficient magnitude of those biases.

The lack of equilibrium feedback in the (perceived) precision of the price greatly simplifies the specification of the equilibrium system. Using this linear structure we arrive at a characterization of the candidate equilibrium, in closed form for the linear case, implicitly for all regular economies.

Proposition 1. *If an equilibrium exists, then it satisfies*

$$\beta = \frac{1}{\rho}\left[\hat{\tau}_s\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right) - \hat{\tau}_p(\beta)\left(\frac{2\Lambda_1 + \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right)}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right] \tag{3.3}$$

$$\gamma = \frac{\tau_0 + \hat{\tau}_s + \hat{\tau}_p(\beta)}{\hat{\tau}_s + \hat{\tau}_p(\beta)\left(1 + \frac{\Lambda_p}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)}\beta \tag{3.4}$$

Our main result in this section characterizes the condition for equilibrium existence in terms of the behavior of the precision of the price in a setting where traders only use an infinitesimal amount of their private information. For linear economies, we can plug (3.2) into (3.3) and obtain a (unique) candidate equilibrium

$$b(\vartheta) = \frac{\tau_s}{\rho}\left[\Sigma_{11}^{-1}\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right) - \frac{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)}{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}\left(2\Lambda_1 + \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right)\right)\right]. \tag{3.5}$$

Though the expression seems analytically cumbersome it simplifies considerably when restricting attention to specific biases (see Section 3.1 for examples). Crucially, (3.5) maps each linear economy ϑ into a single number, representing the only possible equilibrium loading on private information.

Theorem 1. *An equilibrium exists in the economy $\vartheta = (\Sigma, \Lambda)$ if and only if*

(1) *the market clearing rule is perceived with (cognitive) noise, $\Lambda_z > 0$, or*
(2) *in the linear economy ($\Lambda_z = 0$) if the candidate loading*
  (a) *is positive,*
$$b(\vartheta) > 0,$$
  or
  (b) *is negative,*
$$b(\vartheta) < 0,$$
  and

$$\Lambda_2 \left(\Sigma_{22} - \mathbf{1}^T \Sigma_{12}\right) \Lambda_2^T < 0. \tag{3.6}$$

Following Definition 2, an equilibrium is a solution to the matching coefficients equations that satisfies the traders' second-order condition. The matching coefficients equations always have a solution, so the question of existence reduces to the second-order condition which, using (3.3)-(3.4) can be rearranged to

$$\frac{\kappa(\vartheta)}{\beta} + \rho \mathbb{V}(\nu | s_i, p) > 0$$

where $\kappa(\vartheta)$ is a positive constant. Since $\mathbb{V}(\nu | s_i, p)$ is always positive, any positive solution to the matching coefficient equation is a valid equilibrium. Whenever $\Lambda_z > 0$ the matching coefficient equation has a positive solution, which is therefore an equilibrium. Intuitively this is because $\Lambda_z > 0 \Rightarrow \lim_{\beta \to 0} \hat{\tau}_p(\beta) = 0$: whenever the market clearing equation is perceived with noise, the volume of informed trading can shrink enough that prices contain almost no information and it is therefore sufficient that the constant term in (3.3) is positive, i.e. that $1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p} > 0$. Notice that in the Kyle economy (rational but with $\Lambda_z > 0$) the condition reduces to $1 > \frac{1}{N-1}$.

If $\Lambda_z = 0$, instead $\hat{\tau}_p$ is no longer endogenous and there is a unique solution (3.5) to the matching coefficient equation: if the solution is negative it has to be large in absolute value so that the positive variance term can dominate, and (3.6) ensures that this is the case.

Notice that, if $\Sigma_{12} = 0$ then the condition (3.6) for a negative candidate b to be an equilibrium reads

$$\Lambda_2 \Sigma_{22} \Lambda_2^T < 0, \tag{3.7}$$

which can never be satisfied as Σ_{22} is a positive semi-definite matrix. Therefore, an immediate corollary of Theorem (1) is that

Corollary 1. *In linear economies where traders have no misperception about the correlation of their individual signal with the other traders' signals ($\Sigma_{12} = 0$), there is an equilibrium if and only if the candidate loading on private information $b(\vartheta)$ is positive.*

In other words, because linear economies lack the adjustment margin as $\hat{\tau}_p(\beta)$ is constant, existence requires that traders are sufficiently biased in their perception of the information environment or of the market clearing, as we will explore shortly in a series of examples. Notice that the rational economy induces $b\left(\vartheta^R\right) = -\frac{N}{N-1}\frac{\tau_s}{\rho} < 0$ and (3.7) further simplifies to $\mathbf{1}I\mathbf{1} < 0$, which does not hold, reaffirming that it cannot sustain existence in the linear (no-noise-trader) case.

3.1. **Examples**[4].

3.1.1. *Overconfidence.* The model with overconfidence deviates from the rational model only in setting $\Sigma_{11}^{-1} = \xi > 1$. A version of this model constituted the static benchmark in Kyle, Obizhaeva, et al. (2018). Notice that this restriction satisfies the hypothesis of Corollary 1, hence an equilibrium exists if and only if the candidate loading on private information is positive. By direct substitution in (3.5), we obtain

$$b_\xi = \frac{\tau_s}{\rho}\left[\frac{N-2}{N-1}\xi - 2\right]$$

and therefore we have existence if and only if

$$\xi > 2\frac{N-1}{N-2}. \tag{3.8}$$

In particular, with bilateral trade, overconfidence alone is not capable of restoring existence; for large N, traders need to perceive their signal to be at least twice as informative as it truly is. Compared to the rational candidate, the overconfident agent puts excessive weight on his private signal.

To see why a modicum of overconfidence is not enough, note that *informationally* a rational agent wants to load on all signals equally, which can be achieved through the price p. In addition, each agent wants to hold back on using his own signal relative to the informativeness benchmark due to *market impact.* As a result, the desired weight on the private signal is strictly negative in the rational benchmark and remains so with slight overconfidence. In equilibrium, with small overconfidence

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[4]All derivations from the general case to the specific examples are straightforward but rather algebraically tedious. We provide detailed calculations for the example of information projection, in Appendix A.1.

the loading unravels as all traders want to put a greater weight on the price than on their signal. To overcome this market power effect, overconfidence needs to be sufficiently strong, indeed, at least a factor of two is required.

Remark 1. Note that the above reasoning does not support equilibrium existence without noise traders in a rational model with heterogeneous traders. While market participants whose information is at least twice as precise as that of the average trader would be willing to use it sufficiently, in a rational model there has to be at least one trader with below average precision, from where any candidate equilibrium unravels. It is only in a behavioral/non-common prior model that every trader can believe that they are at least twice as precise as their average competitor.

Remark 2. Caballé & Sákovics (2003) study a setting where individual overconfidence and the belief about the overconfidence of others are separate, while both coincide in our setting. They study the risk-neutral limit with noise traders, so existence for low overconfidence is guaranteed. They show that when agents believe that they trade *against* excessively overconfident traders, existence is threatened: In this case, the only candidate equilibrium is inverted ($\beta < 0$), which can never be an equilibrium in the risk-neutral limit.

3.1.2. *Under-Appreciation of the Information of Others.* Consider the setting with $\Lambda = \begin{pmatrix} 1 & , \mathbf{1}, & 0, & 0 \end{pmatrix}$, $\Sigma_{11} = 1, \Sigma_{12} = 0$ and $\Sigma_{22} = I\delta\left(1-\phi\right) + \mathbf{1}^T \delta\phi\mathbf{1}$, where the parameter δ captures dismissal of others' precision and ϕ is correlation delusion; $\delta = 1$, $\phi = 0$ represent the rational benchmark. Since still $\Sigma_{12} = 0$, Corollary 1 applies and existence is characterized by a positive loading on private information.[5] In this setting, equation (3.5) simplifies to

$$b_{\delta,\phi} = \frac{\tau_s}{\rho}\left[\frac{N-2}{N-1} - \frac{2}{\delta\left(1+\left(N-2\right)\phi\right)}\right]$$

which is positive if and only if

$$\delta\left(1+\left(N-2\right)\phi\right) > 2\frac{N-1}{N-2}$$

that is, if δ and ϕ are sufficiently large. If traders dismiss the information possessed by their competitor, either because it is heavily correlated (large ϕ) or because it is poor to begin with (large δ), then they are willing to load on their private signal and an equilibrium exists. More generally, we can consider the subjective precision of the price, $\hat{\tau}_p$, in effective units of other informed traders. That is, we parametrize $\hat{\tau}_p = \left(n-1\right)\tau_s$ where n is the equivalent number of informed traders. Then the

[5] Misperceptions of correlation are studied and documented in the literature (Ellis & Piccione, 2017; Enke & Zimmermann, 2019). The model without correlation delusion ($\phi = 0$) is essentially equivalent to the model with overconfidence analyzed in the previous section since, by suitably adjusting τ_0, the model can be reparametrized in terms of *relative* overconfidence $\xi \cdot \delta$ alone.

candidate equilibrium is

$$b_n = \frac{\tau_s}{\rho}\left[\frac{N-2n}{N-1}\right]$$

which is positive if and only if $n < \frac{N}{2}$. Hence, to restore existence, we require that traders think that the price contains independent signals of fewer than half of the other traders.

3.1.3. *Information Projection and Inverted Equilibria.* Suppose the only bias in information processing is information projection, that is, given s_i, trader i thinks that trader j observes signal

$$s_j = \alpha s_i + (1-\alpha)\nu + \sqrt{(1-\alpha^2)}\epsilon_j$$

which fits into our general framework once we let $\Sigma_{11} = 1$, $\Sigma_{22} = I\left(1-\alpha^2\right) + \mathbf{1}^T\alpha^2\mathbf{1}$ and $\Sigma_{12} = \alpha\mathbf{1}$ for $\alpha \in (0,1)$.[6] As for the market clearing rule, we maintain $\Lambda_1 = \Lambda_2 = \mathbf{1}$, $\Lambda_p = 0$ and consider both the case with and without perceived noise. Henceforth, we consider projective economies $\vartheta_{\alpha,\Lambda_z}$ described by the projection parameter α and the perceived noise Λ_z. Detailed derivations are relegated to Appendix A.1.

In the linear model, where Corollary 1 does not apply and we have to check for potential inverted equilibria, the residual perceived precision contained in the price is

$$\hat{\tau}_p = \frac{(1-\alpha)(N-1)}{1+\alpha}\tau_s.$$

If $\alpha = 0$ we are back to the linear rational benchmark $\hat{\tau}_p = (N-1)\tau_s$, while if $\alpha = 1$ prices contain no residual information since the signals of others simply replicate the private signal. Using (3.3), we get the candidate equilibrium

$$b_\alpha = -\frac{\tau_s}{\rho}\left[\frac{N+(N-2)^2\alpha}{(N-1)(1+\alpha)}\right] \tag{3.9}$$

which is always negative. Using the sufficient condition (3.6), we get that (3.9) constitutes an equilibrium if and only if $\alpha > \frac{1}{N-2}$. There is an equilibrium in a market with (sufficiently) projective traders even in the absence of noise traders, but this equilibrium is inverted: traders load negatively on their private information (and positively on the price).

Let us now consider the model where there is exogenous $\Lambda_z > 0$. Because $\hat{\tau}_p$ is endogenous, (3.3) now defines a candidate equilibrium only implicitly as a solution

[6] Information projection is introduced by Madarasz (2012). Our parametrization ensures that the signals remain jointly Gaussian and that their ex-ante precision is unaffected by projection.

to the cubic equation

$$\beta = \frac{\tau_s}{\rho}\left[\left(1-\frac{1}{N-1}\right)-\frac{2+(N-1)\,\alpha\left(1-\frac{1}{N-1}\right)}{(1+\alpha)+\frac{\Lambda_z}{\beta^2}}\right]. \tag{3.10}$$

Equation (3.10) always admits a real positive solution, which per Theorem 1 then constitutes an equilibrium; as for its negative root(s), the SOC is satisfied at those candidate equilibria if and only if

$$|\beta| > \Lambda_z\sqrt{\frac{(N-2)\,\alpha-1}{(N-1)\,(1-\alpha)}}. \tag{3.11}$$

Combining (3.10) and (3.11) defines a region in the (α, Λ_z) space where a projective (negative β) equilibrium exists. Studying this region, and comparing the properties of (eventual) projective equilibria with the noise-driven equilibrium (positive roots of (3.10)) gives important insights.

Remark 3. [Existence of projective equilibria] Recall that for $\Lambda_z = 0$ a projective equilibrium exists as long as $\alpha > \frac{1}{N-2}$. On the contrary, there is a threshold $\bar{\Lambda}$ such that if $\Lambda_z > \bar{\Lambda}$ then for no $\alpha \in (0,1)$ the economy $\vartheta_{\alpha,\Lambda_z}$ admits a projective equilibrium. In general, for $\Lambda_z > 0$ a projective equilibrium exists only for intermediate degrees of projection: condition (3.11) still requires $\alpha > \frac{1}{N-2}$, but a projective equilibrium now also fails for α close to 1, where the residual information that traders perceive in the price vanishes and is dominated by the perceived noise. The set of α for which $\vartheta_{\alpha,\Lambda_z}$ admits a projective equilibrium shrinks as Λ_z grows and is empty for $\Lambda_z > \bar{\Lambda}$.

Remark 4. [Multiplicity and selection local to no-noise]. Fix some $\alpha^\star > \frac{1}{N-2}$ and a small $\Lambda_z^\star$. In an open ball around $(\alpha^\star, \Lambda_z^\star)$ there are two equilibria: the noise trader (small positive β^N) and the projective one (large negative β^P). Both equilibria are continuous in the ball, as is the cubic equation. However, on the boundary where $\Lambda_z = 0$, only the projective equilibrium exists. Moreover, for each α in the ball,

$$b_\alpha = \lim_{\Lambda_z \to 0} \beta^P\left(\vartheta_{\alpha,\Lambda_z}\right) < 0 = \lim_{\Lambda_z \to 0} \beta^N\left(\vartheta_{\alpha,\Lambda_z}\right),$$

which suggests that it is more natural to select the projective equilibria around $\Lambda_z = 0$, and that the approach of adding projection in a (noise trader) setting where an equilibrium already exists might lead to misleading conclusions. This is because the instrument (noise trading) employed to ensure existence also renders the bias irrelevant at the candidate equilibrium, while the bias itself can lead to existence.

3.1.4. *Misperceptions of Market Impact.* Now consider traders who correctly perceive the information environment (Σ is the identity matrix), but misperceive their position within the market. Instead of the correct market clearing rule $\sum_{j\in[N]} x_j = 0$, each trader i believes that the price is determined according to $m_1 x_i + m_2 \sum_{j\neq i} x_j = 0$, where $m = \frac{m_1}{m_2}$ represents an individual's perception of the relative importance of one's own demand (relative to that of others) in determining the market price.[7]

Since traders correctly perceive the information environment, the information they extract from the price is unaffected by m: $\hat{\tau}_s = \tau_s$ and $\hat{\tau}_p = (N-1)\,\tau_s$, as in the rational economy. The relative loadings are therefore accurate, as matching coefficients pins down the coefficient

$$\beta = \frac{\tau_s}{\rho}\left(1 - m\frac{2N-1}{N-1}\right)$$

If the agent ignores his own market impact, i.e. if $m = 0$, we have $\beta = \frac{\tau_s}{\rho}, \gamma = \frac{\frac{\tau_0}{N}+\tau_s}{\rho}$ and an equilibrium exists in which each trader correctly extracts the information available but disregards the fact that his actions move the market against him. More generally, equilibrium existence requires

$$m < \frac{N-1}{2N-1} < \frac{1}{2}.$$

In particular, underestimating the impact of other traders, m_2, is not helpful but harmful for existence. In the linear model, the information content of the price is independent of market clearing parameters and m_2 affects the equilibrium solely by amplifying the perceived market power.

The case of $m_2 = 0$, by contrast, is fundamentally different as it makes the economy non-regular. For this case, let us reintroduce a perceived market depth $\Lambda_p > 0$, without which the perceived market clearing rule would no longer admit a solution. The price then has no subjective information content and the analysis is equivalent to the case $\hat{\tau}_p = 0$. The candidate solution is

$$\beta = \left(1 - \frac{\Lambda_1}{\Lambda_p}\right)\frac{\tau_s}{\rho}$$
$$\gamma = \left(1 - \frac{\Lambda_1}{\Lambda_p}\right)\frac{\tau_0+\tau_s}{\rho}$$

and to ensure existence, we require that the residual market demand is not too steep relative to individual trader's impact, i.e. $\Lambda_p > \Lambda_1$.[8]

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[7] We consider a restriction of (2.4), letting $\Lambda_1 = m_1$ and $\Lambda_2 = m_2\mathbf{1}^T$ to capture with a single parameter individuals' relative misperception of market power. $\Lambda_p = 0$ is assumed throughout this example, except in the case $m_2 = 0$ below, where we note its reintroduction explicitly. Note that market *impact* itself is an endogenous quantity: what traders misperceive is the market clearing *rule* (2.4), a primitive of their subjective model, and the impact λ they perceive follows from it via (2.5).

[8] In Appendix B.1, we show that this intuition generalizes to all non-regular economies.

3.2. **Comparative Statics of Existence.** Recall that existence is guaranteed in non-linear economies (Theorem 1). In this section we turn to the question of what biases relax or make more stringent the existence condition for linear economies. From now on, we will work with the model restrictions introduced in the examples and adopt the parametrization $\Sigma_{11} = \xi$, $\Sigma_{22} = I\delta(1-\phi) + \mathbf{1}^T\delta\phi\mathbf{1}$, $\Sigma_{12} = 0$, and $\Lambda_1 = m_1$, $\Lambda_2 = m_2\mathbf{1}$. The approach extends to the general setting considered in the previous sections, though $\Sigma_{12} \neq 0$, i.e. perceived correlation between individual and others' signals, induces interdependence of information and market clearing misperceptions and thus makes it impossible to summarize the existence comparative statics in two functions H, E, one pertaining to misperceptions of the information environment, one pertaining to misperceptions of the market clearing equation.

Define the Hubris and Egocentrism functions $E, H : \Theta \to \mathbb{R}$, where Θ denotes the set of linear economies ϑ in the restricted parametrization just introduced, as follows[9]

$$H(\vartheta) = \frac{\xi\delta(1 + (N-2)\phi)}{(N-1)}, \qquad E(\vartheta) = \frac{1 - \frac{m_1}{\Lambda_p + m_2(N-1)}}{2\frac{m_1}{m_2(N-1)}} \tag{3.12}$$

Proposition 2. *Consider two linear economies ϑ, ϑ' and suppose an equilibrium exists in ϑ. Then, if $H(\vartheta') \geq H(\vartheta)$ and $E(\vartheta') \geq E(\vartheta)$, an equilibrium exists in ϑ'.*

Existence, therefore, is determined by two dimensions of the traders' misperception. First, how much a trader overestimates the precision of her private signal relative to the information of others contained in the market price (hubris, H). In particular, economies with more overconfident, dismissive and correlation-delusional traders are conducive for equilibrium existence: An equilibrium exists only if each individual trader thinks he is sufficiently more informed than the rest. Second, how much a trader underestimates his impact on the price relative to market depth (egocentrism E). In particular, it is helpful for existence if traders believe that their impact on price formation is small, that the information of others is heavily priced and that the market is deep, either because others strongly react to the price or because there is additional uninformed capacity. Consider indeed the further restriction where $\Lambda_p = 0$, i.e. there is no uninformed capacity. Then, $E(\vartheta)$ is proportional to $\frac{m_2}{m_1}$, the relative misperception of others' to own market impact.

[9]Proposition 2 extends to general economies under the appropriate redefinition of the hubris and egocentrism functions. In particular, $H(\vartheta) = \frac{\hat{\tau}_s(\vartheta)}{\hat{\tau}_p(\vartheta)}$, $E(\vartheta) = \frac{b_1(\vartheta)}{b_2(\vartheta)}$, substituting (3.2) for the price precision and $b_1 = \frac{2\Lambda_1 + \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right)}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}$, $b_2 = 1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}$ give the Proposition. Such general formulation, however, confounds the intuition of what misperceptions are conducive to existence due to the interaction of information and market impact of the different traders.

In short, an equilibrium exists in an economy of traders who think they are much smarter than anybody else but do not move the price, while existence fails in an economy of traders who want to listen to the price and think the price listens to them.

Another important question is whether a larger number of traders is beneficial for equilibrium existence. On the one hand, more traders increase the information content of the price. On the other hand, it dilutes market power. The second effect dominates.

Proposition 3. *Suppose that $\phi \geq 0$. Then, if ϑ_N admits an equilibrium, then $\vartheta_{N'}$ admits an equilibrium for all $N' > N$.*

The proof of this result is a straightforward algebraic argument that a larger number of traders relaxes the condition $b(\vartheta) > 0$ which, in light of Corollary 1, is the only route to existence. The perceived information content of the price grows as more traders add their private information, but this effect is overpowered by the reduction in market power. The following example illustrates that this result could be overpowered if a bias leads the agent to perceive the precision of the price in a way that is steeply increasing in N.

Example 1 (Perceived Negative Correlation)**.** Suppose that the agent believes that the signals of the other traders are negatively correlated. Note that there is a bound on this perceived correlation, $\phi > -\frac{1}{N-2}$, as there cannot be too many signals of any given pairwise negative correlation. This means that we cannot even perform the analysis for an arbitrary number of traders. Even when correlation is physically possible, it implies that the subjective precision of the price grows quickly in the number of traders, and hence Proposition (3) does not extend to this case. To see why, consider a three trader model with perceived correlation $\phi = -1$ (coinciding with the lower bound at $N = 3$) so that each agent i thinks that $\epsilon_j = -\epsilon_k$ and therefore $s_j + s_k = \nu$. Because of the perfect negative correlation, the price subjectively reveals the fundamental and an equilibrium cannot exist in a regular economy. For $N = 2$, instead, the subjective correlation has no impact, and we can have an equilibrium with a sufficient underestimation of market power. This failure of monotonicity in existence is general in the case of negative ϕ, up to integer constraints.

4. Trade Volume

The question of existence is tightly linked to the question of trade volume. The non-existence problem arises in the benchmark model since no informed trader wishes to trade based on their information, a parametric manifestation of no-trade theorems. Adding noise traders ensures existence and some informed trade, yet the volume of informed trade is closely linked to the volume of noise trading. Given the large and purportedly-informed trade volume observed in practice (Barber & Odean, 2000), this is not necessarily a desirable conclusion.

To address this question and in order to derive further positive implications from our setting, this section analyzes informed trading volume in our model. We consider our model as before, in the fully general specification of Section 2. We now interpret the noise term in the market clearing condition, Λ_z, as a genuine noise trader shock related to liquidity trading, as in the classic literature. This change of interpretation, however, allows us to study the extent to which trading volume is driven by exogenous noise trading compared to the misperceptions we introduce. As we consider a large class of misspecifications and aim to characterize the general consequences of our misperception-based existence mechanism, we focus on the qualitative behavior of the equilibrium as noise trading vanishes or the number of traders grows. Note that $\Lambda_z > 0$ throughout this section, so that an equilibrium always exists (point 1 of Theorem 1).

In equilibrium, market clearing implies that the position of a single trader is $x_i = \beta(s_i - \bar{s}) - \frac{1}{N}\Lambda_z \tau_s^{-1/2} z$, where $\bar{s} = \frac{1}{N}\sum_j s_j$, the sum of two independent Gaussian terms. Her expected trade volume is therefore given by

$$\mathbb{E}[|x_i|] = \sqrt{\frac{2}{\pi}}\sqrt{\tau_s^{-1}\frac{N-1}{N}\beta^2 + \frac{1}{N^2}\Lambda_z^2\tau_s^{-1}} \tag{4.1}$$

Note that trade volume has two components. The first component is driven by the trader's information; the second component is driven by the noise trader shock which has to be absorbed by the informed traders. We hence define the share of informed trading volume as

$$I := \frac{N\sqrt{\mathbb{E}[|x_i|]^2 - \frac{2}{\pi}\frac{1}{N^2}\Lambda_z^2\tau_s^{-1}}}{N\mathbb{E}[|x_i|] + \Lambda_z\tau_s^{-1/2}\mathbb{E}[|z|]} \tag{4.2}$$

which denotes the fraction of total trade volume (the informed traders' components plus the volume contributed by the noise traders themselves) that is driven by information-based trading.

4.1. **Volume when Noise Trading is Small.** Consider an economy ϑ_{Λ_z} with an arbitrary misspecification. We study its equilibrium and trade volume as the standard deviation of the noise trader shock, Λ_z, goes to zero. To this purpose, it

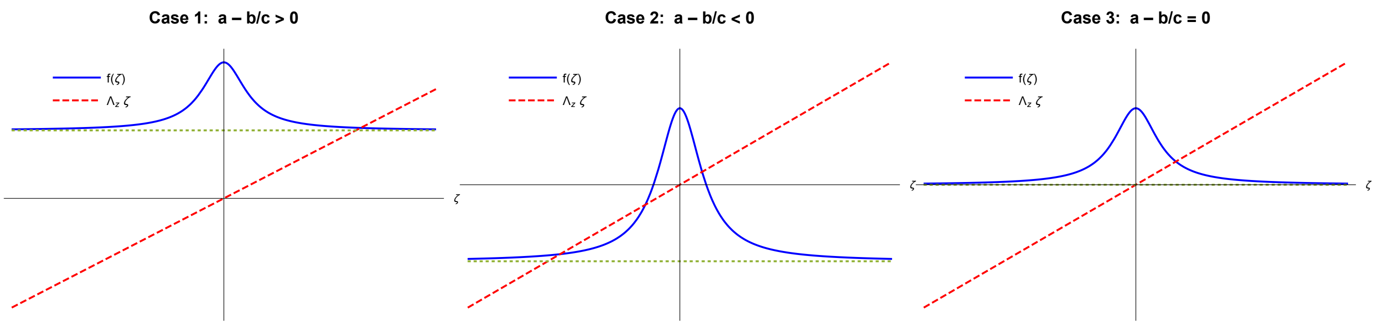

FIGURE 4.1. The limit solution for ζ as $\Lambda_z \to 0$.

is convenient to define the reduced form parameters

$$a = \frac{\tau_s}{\rho}\Sigma_{11}^{-1}\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right) > 0 \tag{4.3}$$

$$b = \frac{\tau_s}{\rho}\left[2\Lambda_1 + \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1 - \frac{\Lambda_1}{\mathbf{1}\Lambda_2^T + \Lambda_p}\right)\right]\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right) \tag{4.4}$$

$$c = \Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T \tag{4.5}$$

Then, the equilibrium map can be written in terms of the normalized weight of the private signal $\zeta = \frac{\beta}{\Lambda_z}$ as

$$\zeta\Lambda_z = a - \frac{b}{c + \zeta^{-2}}$$

Note that the RHS equals a for $\zeta = 0$ and goes to $a - \frac{b}{c}$ as $\zeta \to \pm\infty$. Furthermore, note that in this parametrization, RHS is independent of Λ_z. Hence, finding a solution for small Λ_z can be visualized as the intersection of the two curves in Figure 4.1 as the line rotates towards the x-axis.

Recall that the condition for equilibrium existence without noise is equivalent to $a - \frac{b}{c} > 0$; we henceforth call it the *self-sustaining trade condition.* Since an equilibrium always exists here due to the presence of noise traders, this condition still determines the behavior of the equilibrium as noise trading vanishes. When this condition is satisfied, ζ grows without bounds; equivalently, the use of private information in equilibrium, β does not vanish. Conversely, when this condition is violated, informed trading is tied to noise trading and vanishes together with it. Formally, we have

Proposition 4. *Consider an economy ϑ_{Λ_z}. Then, as $\Lambda_z \to 0$*

1. *informed trading stabilizes ($\beta \to a - \frac{b}{c} + O(\Lambda_z^2)$) if the self-sustaining trade condition holds, $a - \frac{b}{c} > 0$;*
2. *informed trading vanishes at a linear rate ($\beta \to \Lambda_z/\sqrt{\frac{b}{a} - c}$) if it is violated, $a - \frac{b}{c} < 0$; and*

(3) *informed trading vanishes sub-linearly* ($\beta \to \left(\frac{a}{c}\right)^{1/3} \Lambda_z^{2/3} + O(\Lambda_z^{4/3})$) *if it is violated with equality* $a - \frac{b}{c} = 0$.

This result translates directly into the informed volume share. We have

$$I = \frac{|\zeta|}{|\zeta| + 2\sqrt{\frac{1}{N(N-1)}}}$$

If the self-sustaining trade condition is violated strictly, the informed share of volume is bounded. As noise trading vanishes, so does informed trading. In particular, we would expect little (putatively) informed trade in a context with little noise trading. If the existence condition is satisfied, informed trading volume is not bound to the noise volume. As noise trading vanishes, the share of informed trading in volume grows without bounds. This corresponds to a market in which (subjectively) informed traders trade on their information in great excess of true noise trading. Finally, in the knife-edge case in which the condition is violated at equality, informed trading vanishes as noise trading vanishes, but much slower, such that the share of informed trading does grow.

4.2. **Volume per Trader.** We can also ask what happens when more potentially informed traders enter a market with a fixed noise trading activity. How does the trading volume, both total and per trader (4.1) evolve?

To answer this question, we return to the symmetric parametrization of Section 3.2, where we normalize $\Lambda_z = 1$ and assume that there is no projection ($\Sigma_{12} = 0$). Furthermore, we need to make an assumption about how the perceived covariance of the signals of others changes as the economy grows. Let us assume that the entries satisfy $\frac{1}{N}\sum_{i,j}(\Sigma_{22})_{i,j} \to \sigma > 0$. In particular, this condition ensures that subjectively, the average signal of the other traders does satisfy a law of large numbers and converges to the state.[10]

To express our conditions in this limit case, write the limit of the reduced form parameters (4.3)-(4.5) as $N \to \infty$ as a_∞ and $\frac{b_\infty}{c_\infty}$.[11] Then

Proposition 5. *Consider a symmetric economy without projection and* $\frac{1}{N}\sum_{i,j}(\Sigma_{22})_{i,j} \to \sigma > 0$ *as* $N \to \infty$. *Then,*

(1) *informed trading stabilizes* ($\beta \to a_\infty - \frac{b_\infty}{c_\infty} + O(N^{-1})$) *if* $a_\infty - \frac{b_\infty}{c_\infty} > 0$;
(2) *informed trading vanishes at rate* $N^{-1/2}$ ($\beta \to \frac{1}{\sqrt{N}} / \sqrt{\frac{b_\infty}{a_\infty} - c_\infty}$) *if* $a_\infty - \frac{b_\infty}{c_\infty} < 0$; *and*
(3) *informed trading vanishes slowly* ($\beta \to CN^{-\frac{1}{3}} + O(N^{-2/3})$) *if* $a_\infty - \frac{b_\infty}{c_\infty} = 0$, *for the constant* $C = \left(\frac{a_\infty}{c_\infty}\right)^{1/3}$.

[10] The more general case can be treated in a similar way.

[11] Even though some of these separate expressions do not converge, we make sure to only use them in expressions that are meaningful.

When the self-sustaining trade condition holds, each trader's use of private information stabilizes in the limit, so that per-trader informed volume remains bounded away from zero and aggregate informed volume grows linearly with N, dwarfing the fixed noise trading volume. Entry of (subjectively) informed traders thus generates its own volume: the market does not need additional liquidity trading to absorb additional speculators. When the condition fails, by contrast, the fixed pool of noise trading is competed over by an ever larger crowd: informed trade per capita vanishes at rate $N^{-1/2}$ and aggregate informed volume grows only as $\sqrt{N}$. Misperceptions satisfying the self-sustaining trade condition therefore not only ensure existence without noise, they also decouple the scale of informed trading from the scale of liquidity trading, consistent with markets in which putatively informed volume vastly exceeds plausible hedging demand.

5. Conclusion

We investigate the behavioral foundations of informed trade. We augment the classic Kyle model to encompass a variety of biases in how traders perceive the information environment, as well as the market clearing condition which determines both the way individuals think their actions affect the price and what information the price conveys. The classic noise trader model generates an adjustment margin of the endogenous precision of prices which ensures equilibrium existence making it possible for the private signal to be precise enough relative to the information contained in prices. We develop within our general framework a reinterpretation of the noise traders' shock as a cognitive error in conceptualizing the pricing functional; this model results in the same equilibrium behavior, but features different comparative statics of, say, price efficiency.

Within the more general framework, this cognitive noise model represents only one very specific parametric restriction that guarantees existence. Moreover, existence comes at the cost of breaking the linearity that otherwise characterizes the model and considerably simplifies its analysis. The cost of conducting the analysis in the linear restriction of the general framework is, instead, that substantial biases are needed for existence: no equilibria exist in a neighborhood of the rational model. Instead, our results on trade volume support a conditional reading: if observed, putatively informed, trading volume is large relative to plausible levels of noise trading, then through the lens of this class of models the biases of the traders must satisfy the self-sustaining trade condition. Indeed, although equilibria exist for arbitrarily small noise trader shock, those close to rationality ($\Lambda_z \approx 0$) entail essentially no-trade ($\beta \approx 0$) as the informed trading volume needs to shrink to match the relative magnitude of noise trader demand. Generating substantial informed volume then requires either substantial noise trading – with the large-or-correlated-noise problems discussed in the introduction, – or systematic biases strong enough to sustain trade on their own. The self-sustaining trade condition thus demarcates between two views of what animates markets: markets animated by large, deliberate, correlated noise trading that sophisticated participants nevertheless cannot predict, and markets animated by traders who believe they have an informational edge, and who may overestimate it due to systematic misperceptions.

We show that existence in the linear model requires instead that traders are strongly hubristic, i.e. they perceive the precision of their signal (relative to the price) to be large, and/or not very egocentric, i.e. think that their actions (relative to the actions of others) does not have a significant impact on the price. Biases that make traders overestimate the precision of their information conduce to existence, as they make them more willing to bet. By contrast, biases that make traders overestimate their impact on the market are detrimental for existence, since thinking that the price will move strongly against them makes them more reluctant to act. In finite economies, more traders dilute market power and therefore relax the equilibrium existence condition: adding traders never breaks existence. In ongoing work, we show that this result does not extend to a limit economy with countably many traders. Instead, existence relies on a property, limit uncertainty, which requires that traders think their private signal is informative about the fundamental even conditioning on the infinite collection of competitors' signals. In particular, misperceptions of market power are powerless in the limit economy which instead requires that traders perceive some positive correlation in their competitors' information. We also

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Appendix A. Appendix: Proofs

Proof of Lemma 1. Recall that, subjectively, $\begin{pmatrix} s_i \\ \boldsymbol{s_{-i}} \end{pmatrix} \sim \mathcal{N}\left(\boldsymbol{\nu}, \tau_s^{-1} \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{12}^T & \Sigma_{22} \end{pmatrix}\right)$.
Then,

$$\begin{pmatrix} s_i \\ \Lambda_2 \cdot \boldsymbol{s_{-i}} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} \nu \\ \Lambda_2 \cdot \boldsymbol{\nu} \end{pmatrix}, \tau_s^{-1} \begin{pmatrix} \Sigma_{11} & \Lambda_2 \Sigma_{12}^T \\ \Lambda_2 \Sigma_{12}^T & \Lambda_2 \Sigma_{22} \Lambda_2^T \end{pmatrix}\right)$$

using the conditional distribution of jointly normal random variables we obtain

$$\begin{aligned} \Lambda_2 \cdot \boldsymbol{s_{-i}} | s_i &\sim \mathcal{N}\left(\Lambda_2 \boldsymbol{\nu} + \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1}(s_i - \nu), \tau_s^{-1} \Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12}\right) \Lambda_2^T\right) \\ &= \mathcal{N}\left(\left(\mathbf{1}\Lambda_2^T - \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1}\right) \nu + \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1} s_i, \tau_s^{-1} \Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12}\right) \Lambda_2^T\right) \end{aligned}$$

and

$$\frac{\Lambda_2 \cdot \boldsymbol{s_{-i}} - \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1} s_i}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1}\right)} \sim \mathcal{N}\left(\nu, \tau_s^{-1} \frac{\Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12}\right) \Lambda_2^T}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1}\right)^2}\right)$$

and finally,

$$\begin{aligned} \frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta} p_i - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} &= \frac{\Lambda_2 \boldsymbol{s}_{-i} - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T s_i + \frac{\Lambda_z}{\beta} \tau_s^{-1/2} z}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} \\ &\sim \mathcal{N}\left(\nu, \tau_s^{-1} \frac{\Lambda_2 \left(\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12}\right) \Lambda_2^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Lambda_2 \Sigma_{12} \Sigma_{11}^{-1}\right)^2}\right) \end{aligned}$$

and conditionally independent of s_i, proving expressions (2.8)-(2.9)-(2.10). □

Proof of Proposition 1. The candidate loadings in the linear equilibrium $x_i = \beta s_i - \gamma p$ are obtained matching coefficient of the best response function $x_i = \frac{\mathbb{E}(\nu|s_i,p) - p_i}{2\lambda_i + \rho \mathbb{V}(\nu|s_i,p)}$, substituting for p_i, λ_i their expressions (2.6) and (2.5), and the misspecified expectations and variances derived using equations (2.8)-(2.9)-(2.10) in Lemma 1.

$$\begin{aligned} \frac{\mathbb{E}\left(\nu|s_i,p\right) - p_i}{2\lambda_i + \rho \mathbb{V}\left(\nu|s_i,p\right)} &= \frac{\frac{s_i \hat{\tau}_s + \hat{p}_i \hat{\tau}_p}{\hat{\tau}_p + \hat{\tau}_s + \tau_0} - p_i}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}} \\ &= \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s + \hat{\tau}_p + \tau_0} s_i + \frac{\hat{\tau}_p}{\hat{\tau}_s + \hat{\tau}_p + \tau_0} \frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta} p_i - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} - p_i}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}} \end{aligned}$$

$$= \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s + \hat{\tau}_p + \tau_0} s_i + \frac{\hat{\tau}_p}{\hat{\tau}_s + \hat{\tau}_p + \tau_0} \frac{\frac{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)\gamma}{\beta}\left(p - \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}(\beta s_i - \gamma p)\right) - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T s_i}{\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1} \Sigma_{12} \Lambda_2^T} - \left(p - \frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}(\beta s_i - \gamma p)\right)}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s + \hat{\tau}_p + \tau_0}}$$

collecting the loadings on s_i, p we obtain

$$\beta = \frac{\frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0} - \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} + \frac{\beta\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}} \tag{A.1}$$

and

$$-\gamma = \frac{\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\frac{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)\gamma}{\beta}\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - \left(1+\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{2\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}}.$$

dividing the two, we obtain

$$\frac{\beta}{\gamma} = -\frac{\frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0} - \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} + \frac{\beta}{\gamma}\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}}{\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)\frac{\beta}{\gamma}} - \left(1+\frac{\Lambda_1}{\gamma\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}$$

$$\frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\mathbf{1}\Lambda_2^T+\Lambda_p+\Lambda_1}{\left(\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)} - \frac{\beta}{\gamma} = \frac{\hat{\tau}_p}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}\frac{\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} - \frac{\hat{\tau}_s}{\hat{\tau}_s+\hat{\tau}_p+\tau_0}$$

which can be rearranged to yield (3.4). Substituting this in (A.1) gives also (3.3).

□

Proof of Theorem 1: After substituting for candidate equilibrium β, γ obtained in Proposition 1, the SOC of the trader's problem reads

$$2\frac{\Lambda_1}{\beta\frac{\hat{\tau}_s+\hat{\tau}_p+\tau_0}{\hat{\tau}_s+\hat{\tau}_p\frac{\mathbf{1}\Lambda_2^T+\Lambda_p-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}}\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)} + \frac{\rho}{\hat{\tau}_p+\hat{\tau}_s+\tau_0} > 0$$

Multiplying through by $\frac{\hat{\tau}_p+\hat{\tau}_s+\tau_0}{\rho} > 0$ and substituting for $\rho\beta$ from (3.3), this is equivalent to

(A.2)

$$2\frac{\Lambda_1\left(\hat{\tau}_s+\hat{\tau}_p\frac{\mathbf{1}\Lambda_2^T+\Lambda_p-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)}{\left[\hat{\tau}_s\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right) - \hat{\tau}_p\left(\frac{2\Lambda_1+\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right]\beta\left(1+\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)}\right)} + 1 > 0$$

Note that if $\beta > 0$, the SOC is satisfied. Otherwise, the denominator above is negative, and we need to flip the inequality. Lengthy but straightforward manipulations show that (A.2) is equivalent to

$$\Lambda_1\hat{\tau}_s - \Lambda_1\hat{\tau}_p\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T} < -\left[\hat{\tau}_s - \hat{\tau}_p\left(\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right]\left(\mathbf{1}\Lambda_2^T+\Lambda_p\right)$$

$$\Leftrightarrow \left(\mathbf{1}\Lambda_2^T+\Lambda_p+\Lambda_1\right)\left[\hat{\tau}_s - \hat{\tau}_p\left(\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)\right] < 0 \tag{A.3}$$

Using $\left(\mathbf{1}\Lambda_2^T + \Lambda_p + \Lambda_1\right) > 0$ and substituting (3.2) for $\hat{\tau}_p$, the inequality simplifies to

$$\left(1 - \frac{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)\Sigma_{12}\Lambda_2^T}{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T}\right)\Sigma_{11}^{-1} < 0$$

and in turns to $\Lambda_2\left(\Sigma_{22} - \mathbf{1}^T\Sigma_{12}\right)\Lambda_2^T < 0$, which is equation (3.6) in the text.

□

Proof of Proposition 2. Recall from now we are working in the restricted economies $\Sigma_{11} = \xi$, $\Sigma_{22} = I\delta(1-\phi) + \mathbf{1}^T\delta\phi\mathbf{1}$, $\Sigma_{12} = 0$, and $\Lambda_1 = m_1$, $\Lambda_2 = m_2\mathbf{1}$. Since Corollary 1 applies to such restriction, an equilibrium exists if and only if (3.5) is positive. Notice that (3.5) is equivalent to

$$\left(1 - \frac{m_1}{\Lambda_p + m_2(N-1)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + (N-2)\phi\delta} > 0$$

$$\iff \frac{\left(1 - \frac{m_1}{\Lambda_p + m_2(N-1)}\right)}{2\frac{m_1}{m_2}} > \frac{1}{\xi\delta\left[1 + (N-2)\phi\right]}$$

using definitions (3.12), the latter condition is equivalent to

$$H(\vartheta)E(\vartheta) > 1,$$

from which the statement follows. □

Proof of Proposition 3. In the non-linear case there is always an equilibrium so the statement is vacuous. In the linear case, fix a misperceived economy ϑ and suppose an equilibrium exists with N traders. By Corollary 1, this implies that $b_N(\vartheta) > 0$. Both terms of b_N are nondecreasing in the number of traders: the first because $\frac{m_1}{\Lambda_p + m_2(N-1)}$ is decreasing in N, the second because $\phi \geq 0$. Whenever $N' > N$ then

$$\begin{aligned} b_{N'}(\vartheta) &= \left(1 - \frac{m_1}{\Lambda_p + m_2(N'-1)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + (N'-2)\phi\delta} \\ &> \left(1 - \frac{m_1}{\Lambda_p + m_2(N-1)}\right)\xi\tau_s - 2\frac{m_1}{m_2}\frac{\tau_s}{\delta + (N-2)\phi\delta} = b_N(\vartheta) > 0 \end{aligned}$$

and, by Theorem 1, an equilibrium exists with N' traders. □

Proof of Proposition 4: Recall that we can write the equilibrium map as

$$\zeta\Lambda_z = f(\zeta), \qquad f(\zeta) := a - \frac{b}{c + \zeta^{-2}}.$$

Note that $f(0) = a > 0$ and $\lim_{\zeta\to\infty} f(\zeta) = a - b/c$. As $\Lambda_z \to 0$, finding an equilibrium corresponds geometrically to intersecting the curve $y = f(\zeta)$ with a line $y = \zeta\Lambda_z$ that rotates toward the horizontal axis.

Case (i): $a - b/c > 0$. Since $f(\zeta) \to a - b/c > 0$ as $\zeta \to \infty$, the intersection $\zeta^*(\Lambda_z) \to \infty$. For large ζ:

$$f(\zeta) = a - \frac{b}{c}\left(1 + \frac{1}{c\zeta^2}\right)^{-1} = a - \frac{b}{c} + \frac{b}{c^2\zeta^2} + O(\zeta^{-4}).$$

Setting $\zeta = (a - b/c)/\Lambda_z + O(\Lambda_z)$ and substituting:

$$\beta = \zeta\Lambda_z = a - \frac{b}{c} + O(\Lambda_z^2).$$

Case (ii): $a-b/c < 0$. Now f has a positive root ζ^* satisfying $a-b/(c+(\zeta^*)^{-2}) = 0$, i.e.,

$$(\zeta^*)^{-2} = \frac{b}{a} - c \quad \Longrightarrow \quad \zeta^* = \frac{1}{\sqrt{b/a - c}}.$$

As $\Lambda_z \to 0$, the intersection converges to this root: $\zeta \to \zeta^* + O(\Lambda_z)$. Hence

$$\beta = \zeta\Lambda_z \to \frac{\Lambda_z}{\sqrt{b/a - c}},$$

which vanishes linearly.

Case (iii): $a - b/c = 0$ *(knife-edge).* Here $b = ac$, so for large ζ:

$$f(\zeta) = a - \frac{ac}{c + \zeta^{-2}} = a \cdot \frac{\zeta^{-2}}{c + \zeta^{-2}} = \frac{a}{c\zeta^2} + O(\zeta^{-4}).$$

The dominant balance $\zeta\Lambda_z \sim a/(c\zeta^2)$ gives $\zeta^3 \sim a/(c\Lambda_z)$, hence

$$\zeta \sim \left(\frac{a}{c}\right)^{1/3} \Lambda_z^{-1/3}.$$

For the next correction, write $\zeta = c_0\Lambda_z^{-1/3} + c_1\Lambda_z^{1/3} + O(\Lambda_z)$ with $c_0 = (a/c)^{1/3}$. Substituting into $\zeta\Lambda_z = f(\zeta)$ and matching $O(\Lambda_z^{2/3})$ terms yields $c_1 = -\frac{1}{3c}(a/c)^{-1/3}$. Therefore

$$\beta = \zeta\Lambda_z = \left(\frac{a}{c}\right)^{1/3} \Lambda_z^{2/3} + O(\Lambda_z^{4/3}),$$

which vanishes sub-linearly. □

Proof of Proposition 5: In the symmetric parametrization of Section 3.2 without projection ($\Sigma_{11} = \xi$, $\Sigma_{12} = 0$, $\Lambda_1 = m_1$, $\Lambda_2 = m_2\mathbf{1}$, $\Lambda_z = 1$), the reduced form parameters (4.3)-(4.5) read

$$a_N = \frac{\tau_s}{\rho\xi}\left(1 - \frac{m_1}{\Lambda_p + m_2(N-1)}\right), \qquad b_N = \frac{2\tau_s}{\rho} m_1 m_2 (N-1), \qquad c_N = m_2^2 \sum_{i,j} (\Sigma_{22})_{i,j}.$$

While b_N and c_N diverge, they do so at a common linear rate, so that the limits in the statement are well defined once normalized by the number of traders:

$$a_\infty := \lim_{N\to\infty} a_N = \frac{\tau_s}{\rho\xi}, \qquad b_\infty := \lim_{N\to\infty} \frac{b_N}{N} = \frac{2\tau_s}{\rho} m_1 m_2, \qquad c_\infty := \lim_{N\to\infty} \frac{c_N}{N} = m_2^2\sigma,$$

where the last limit uses the assumption $\frac{1}{N}\sum_{i,j}(\Sigma_{22})_{i,j} \to \sigma > 0$; this is the sense in which the divergent objects b, c enter only in meaningful combinations: the self-sustaining trade condition involves the ratio $\frac{b_\infty}{c_\infty} = \lim \frac{b_N}{c_N}$, which is normalization-free.[12] All three limits are finite and strictly positive, and

$$a_N = a_\infty + O\left(N^{-1}\right), \qquad b_N = N b_\infty \left(1 + O\left(N^{-1}\right)\right), \qquad c_N = N c_\infty \left(1 + O\left(N^{-1}\right)\right).$$

Note that the scale of Λ_z does not matter as $N \to \infty$, as it only enters proportional to N. We can hence normalize $\Lambda_z = 1$, i.e. $\zeta = \beta$, and the equilibrium map reads

$$\beta = f_N(\beta), \qquad f_N(\beta) := a_N - \frac{b_N}{c_N + \beta^{-2}}.$$

Since $b_N > 0$, f_N is strictly decreasing on $(0,\infty)$ with $f_N(0^+) = a_N > 0$ and $\lim_{\beta\to\infty} f_N(\beta) = a_N - \frac{b_N}{c_N}$, so the map admits a unique positive fixed point β_N, which by Theorem 1 (point 1) is the equilibrium loading. Rearranging the fixed point equation (for N large enough that $a_N > \beta_N$) yields the identity

$$\frac{\beta_N^{-2}}{N} = \frac{b_N/N}{a_N - \beta_N} - \frac{c_N}{N}. \tag{A.4}$$

Case (i): $a_\infty - b_\infty/c_\infty > 0$. Since $\frac{b_N}{c_N+\beta^{-2}} < \frac{b_N}{c_N}$ for all $\beta > 0$,

$$\beta_N = f_N(\beta_N) > a_N - \frac{b_N}{c_N} \longrightarrow a_\infty - \frac{b_\infty}{c_\infty} > 0,$$

so β_N is bounded away from zero and $\beta_N^{-2} = O(1)$. Expanding the fixed point equation using $c_N = \Theta(N)$,

$$\frac{b_N}{c_N + \beta_N^{-2}} = \frac{b_N}{c_N}\left(1 + \frac{\beta_N^{-2}}{c_N}\right)^{-1} = \frac{b_N}{c_N} - \frac{b_N}{c_N^2}\beta_N^{-2} + O\left(N^{-2}\right) = \frac{b_N}{c_N} + O\left(N^{-1}\right),$$

and therefore

$$\beta_N = a_N - \frac{b_N}{c_N} + O\left(N^{-1}\right) = a_\infty - \frac{b_\infty}{c_\infty} + O\left(N^{-1}\right):$$

and informed trading stabilizes.

Case (ii): $a_\infty - b_\infty/c_\infty < 0$. First, $\beta_N \to 0$: Suppose towards a contradiction that $\beta_N \geq \epsilon > 0$ along a subsequence, then $\beta_N^{-2}/c_N \to 0$ and, as in case (i), $\beta_N = a_\infty - \frac{b_\infty}{c_\infty} + o(1) < 0$ eventually, contradicting $\beta_N > 0$. Therefore, we know that $\beta_N \to 0$ and, passing to the limit in (A.4), we have

$$\frac{\beta_N^{-2}}{N} \longrightarrow \frac{b_\infty}{a_\infty} - c_\infty,$$

[12] For the error rates in points 1 and 3 we further assume $\frac{1}{N}\sum_{i,j}(\Sigma_{22})_{i,j} = \sigma + O\left(N^{-1}\right)$, as holds, e.g., for $\Sigma_{22} = I\delta(1-\phi_N) + \mathbf{1}^T\delta\phi_N\mathbf{1}$ with $\phi_N = \frac{\bar{\phi}}{N-1}$, in which case $\sigma = \delta\left(1+\bar{\phi}\right)$. With a slower rate of convergence, the error terms below deteriorate accordingly, while the limits are unaffected.

which is strictly positive precisely because $a_\infty < \frac{b_\infty}{c_\infty}$. Hence $\sqrt{N}\beta_N \to \left(\frac{b_\infty}{a_\infty} - c_\infty\right)^{-1/2}$, that is,

$$\beta_N \to \frac{1/\sqrt{N}}{\sqrt{\frac{b_\infty}{a_\infty} - c_\infty}},$$

and substituting back into (A.4) shows that the relative error is $O\left(N^{-1/2}\right)$: informed trading vanishes at rate $N^{-1/2}$.

Case (iii): $a_\infty - b_\infty/c_\infty = 0$. Here $b_\infty = a_\infty c_\infty$. As in case (ii), $\beta_N \to 0$; but now (A.4) gives $\frac{\beta_N^{-2}}{N} \to \frac{b_\infty}{a_\infty} - c_\infty = 0$, so β_N vanishes strictly slower than $N^{-1/2}$ and, in particular, $\frac{\beta_N^{-2}}{c_N} \to 0$. We may therefore expand f_N around the plateau:

$$\beta_N = \underbrace{a_N - \frac{b_N}{c_N}}_{=O(N^{-1})} + \frac{b_N}{c_N^2}\beta_N^{-2} - \frac{b_N}{c_N^3}\beta_N^{-4} + O\left(\frac{\beta_N^{-6}}{N^3}\right),$$

where $\frac{b_N}{c_N^2} = \frac{a_\infty}{Nc_\infty}\left(1 + O\left(N^{-1}\right)\right)$ using $b_\infty = a_\infty c_\infty$. The dominant balance $\beta_N \sim \frac{a_\infty}{Nc_\infty}\beta_N^{-2}$ gives $\beta_N^3 \sim \frac{a_\infty}{Nc_\infty}$, hence

$$\beta_N \sim \left(\frac{a_\infty}{c_\infty}\right)^{1/3} N^{-1/3}.$$

For the next correction, write $\beta_N = CN^{-1/3} + c_1 N^{-2/3} + O\left(N^{-1}\right)$ with $C = \left(\frac{a_\infty}{c_\infty}\right)^{1/3}$. Substituting into the expansion and matching the $O\left(N^{-2/3}\right)$ terms yields $c_1 = -2c_1 - \frac{a_\infty}{c_\infty^2 C^4}$, i.e. $c_1 = -\frac{1}{3c_\infty C}$, where we used $\frac{a_\infty}{c_\infty C^3} = 1$; the neglected term $a_N - \frac{b_N}{c_N} = O\left(N^{-1}\right)$ only enters at the next order. Therefore

$$\beta_N = CN^{-\frac{1}{3}} + O\left(N^{-2/3}\right), \qquad C = \left(\frac{a_\infty}{c_\infty}\right)^{1/3},$$

which vanishes, but slower than per-trader noise. □

A.1. **Derivations for Economies with Information Projection.** Suppose the only bias in information processing is information projection, that is $\Sigma_{11} = 1$, $\Sigma_{22} = I\left(1-\alpha^2\right) + \mathbf{1}^T\alpha^2\mathbf{1}$ and $\Sigma_{12} = \alpha\mathbf{1}$ for $\alpha \in (0,1)$. As for the market clearing rule, we maintain $\Lambda_1 = 1, \Lambda_2 = \mathbf{1}$, $\Lambda_p = 0$ and consider both the case with and without perceived liquidity traders (Λ_z equal or larger than zero). Under those restrictions, the price precision (3.1) reads

$$\begin{aligned}\hat{\tau}_p^{-1} &= \tau_s^{-1}\frac{\Lambda_2\left(\Sigma_{22} - \Sigma_{12}^T\Sigma_{11}^{-1}\Sigma_{12}\right)\Lambda_2^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\mathbf{1}\Lambda_2^T - \Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T\right)^2} = \tau_s^{-1}\frac{\mathbf{1}\left(I\left(1-\alpha^2\right) + \mathbf{1}^T\alpha^2\mathbf{1} - \mathbf{1}^T\alpha^2\mathbf{1}\right)\mathbf{1}^T + \frac{\Lambda_z^2}{\beta^2}}{\left(\left(N-1\right)\left(1-\alpha\right)\right)^2}\\ &= \tau_s^{-1}\left(\frac{(1+\alpha)}{(N-1)(1-\alpha)} + \frac{\frac{\Lambda_z^2}{\beta^2}}{\left(\left(N-1\right)\left(1-\alpha\right)\right)^2}\right)\end{aligned}$$

In the linear economy $\Lambda_z = 0$ the (exogenous) price precision (3.2) becomes

$$\hat{\tau}_p = \frac{1-\alpha}{1+\alpha}\,(N-1)\,\tau_s$$

and the candidate linear equilibrium (3.5) reads

$$\begin{aligned} b_\alpha &= \frac{\tau_s}{\rho}\left[\left(1-\frac{1}{N-1}\right)-\frac{(N-1)\,(1-\alpha)}{1+\alpha}\left(\frac{2+(N-1)\,\alpha\left(1-\frac{1}{N-1}\right)}{(N-1)\,(1-\alpha)}\right)\right] \\ &= -\frac{\tau_s}{\rho}\left[\frac{N+(N-2)^2\,\alpha}{(N-1)\,(1+\alpha)}\right] \end{aligned}$$

which is always negative, so per Point 2.b of Theorem 1 there is an equilibrium if and only if (3.6) is satisfied. Under projection (3.6) reads

$$\begin{gathered} 1-\frac{(N-1)\,(1-\alpha)}{1+\alpha}\left(\frac{(N-1)\,\alpha}{(N-1)\,(1-\alpha)}\right)<0 \\ \Leftrightarrow \frac{\alpha}{1+\alpha}>\frac{1}{N-1}\Leftrightarrow \alpha>\frac{1}{N-2} \end{gathered}$$

hence a fully projective inverted equilibrium exists in the absence of noise trader under substantial ($\alpha > \frac{1}{N-2}$) projection.

With cognitive noise $\Lambda_z > 0$ then equilibrium is characterized by the cubic (3.10) which always admits a positive solution (Theorem 1 point 1.). We ask whether there are negative solutions that satisfy the second order condition, which in the general case (A.3) reads as

$$\begin{aligned} \hat{\tau}_s-\hat{\tau}_p\left(\frac{\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}{\mathbf{1}\Lambda_2^T-\Sigma_{11}^{-1}\Sigma_{12}\Lambda_2^T}\right)&<0 \\ \Longleftrightarrow\ 1-\frac{1}{\left(\frac{(1+\alpha)}{(N-1)(1-\alpha)}+\frac{\frac{\Lambda_z^2}{\beta^2}}{((N-1)(1-\alpha))^2}\right)}\frac{(N-1)\alpha}{(N-1)(1-\alpha)}&<0 \\ 1-\frac{1}{\left(1+\alpha+\frac{\frac{\Lambda_z^2}{\beta^2}}{(N-1)(1-\alpha)}\right)}(N-1)\alpha&<0 \end{aligned}$$

which now simplifies to

$$\frac{\Lambda_z^2}{\beta^2}<\frac{(N-2)\,\alpha-1}{(N-1)\,(1-\alpha)}$$

delivering, for $\Lambda_z = 0$ the condition above and for $\Lambda_z > 0$ condition (3.11) in the text. Combining (3.11) and (3.10) defines a region in the (α, Λ_z) space where an inverted trade equilibrium exists.

To see why there is a threshold for Λ_z above which no projective equilibria can exist, multiply (3.10) through by $\beta^2\left[(1+\alpha)+\frac{\Lambda_z^2}{\beta^2(N-1)(1-\alpha)}\right]$ and collect terms to write it as $g(\beta)=0$ where

$$g(\beta) \coloneqq (1+\alpha)\beta^3+\frac{\tau_s}{\rho}K\beta^2+\frac{\Lambda_z^2}{(N-1)(1-\alpha)}\beta-\frac{\tau_s}{\rho}\frac{(N-2)\Lambda_z^2}{(N-1)^2(1-\alpha)} \qquad \text{with } K \coloneqq \frac{N+\alpha(N-2)^2}{N-1}.$$

The intercept $g(0)$ is always negative (for $\Lambda_z>0$), consistent with the existence of a positive root. The cubic has an inflection point at $\tilde{\beta}=-\frac{\tau_s}{\rho}\frac{K}{3(1+\alpha)}$, and its first derivative g', an upward parabola minimized at $\tilde{\beta}$, there takes the value

$$g'(\tilde{\beta})=\frac{\Lambda_z^2}{(N-1)(1-\alpha)}-\left(\frac{\tau_s}{\rho}\right)^2\frac{K^2}{3(1+\alpha)}.$$

If $\Lambda_z>\bar{\Lambda}(\alpha)\coloneqq\frac{\tau_s}{\rho}K\sqrt{\frac{(N-1)(1-\alpha)}{3(1+\alpha)}}$ then $g'(\tilde{\beta})>0$ and hence $g'>0$ everywhere, so that g is strictly increasing and (3.10) admits a unique real solution; since $g(0)<0$, this solution is positive. There are then no negative candidates and hence no projective equilibrium. Since $\bar{\Lambda}(\alpha)$ is continuous and bounded on $[0,1]$, the uniform threshold in the text obtains as $\bar{\Lambda}\coloneqq\sup_{\alpha\in(0,1)}\bar{\Lambda}(\alpha)<\infty$. Note also that $\bar{\Lambda}(\alpha)\to 0$ as $\alpha\to 1$: for any fixed $\Lambda_z>0$, projective equilibria fail for α close to 1, where the residual information that traders perceive in the price vanishes and the perceived noise dominates.

Appendix B. Extensions

B.1. **Irregular Economies.** In regular economies (see Definition 1) traders perceive that the market price conveys some information about the state through the signal of others. If this is not the case then the normalization argument to extract information from p_i is invalid as we are dividing by zero. This happens either because other traders do not possess any information to begin with ($\delta = \infty$) or because their action does not have aggregate impact. In this non-regular case, the agent perceives $\hat{\tau}_p = 0$ and the rest of the analysis goes through, yielding

$$\beta = \left(1 - \frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}\right)\frac{\hat{\tau}_s}{\rho}$$

$$\gamma = \left(1 - \frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)}\right)\frac{\tau_0 + \hat{\tau}_s}{\rho}$$

irrespective of the source of non-regularity. If $\Lambda_2 = \mathbf{0}$ then it is needed $\Lambda_p \neq 0$ so that the traders' misperception does not rule out market clearing. Essentially, traders think they are bidding against a fixed (non-stochastic) market clearing condition. Therefore, the only threat to existence is the traders market power. An equilibrium exists if perceived market power is sufficiently small or, conversely, the residual supply sufficiently flat

$$\frac{\Lambda_1}{\left(\mathbf{1}\Lambda_2^T + \Lambda_p\right)} < 1.$$

B.2. **Cursed Economies.** Following the equilibrium notion introduced in Eyster & Rabin (2005), cursed agents form expectations taking a convex combination of the rational (weight $1-\chi$) expectation and the one formed disregarding the information content of prices.

$$\mathbb{E}_\chi\Big[\nu \mid \{p_i, s_i\}\Big] = \chi\mathbb{E}^C\Big[\nu \mid \{p_i, s_i\}\Big] + (1-\chi)\,\mathbb{E}^R\Big[\nu \mid \{p_i, s_i\}\Big]$$

where

$$\mathbb{E}^R\Big[\nu \mid \{p_i, s_i\}\Big] = \frac{\left[\beta s_i + \gamma\,(N-1)\,p_i\right]\tau_s}{\beta\,(\tau_0 + N\tau_s)}$$

is the correct conditional expectation and

$$\mathbb{E}^C\Big[\nu \mid \{p_i, s_i\}\Big] = \frac{\mu_0\tau_0 + s_i\tau_s}{\tau_0 + \tau_s}$$

is obtained disregarding the information content in prices. Similarly,

$$\mathbb{V}_\chi\Big[\nu \mid \{p_i, s_i\}\Big] = \chi\mathbb{V}^C\Big[\nu \mid \{p_i, s_i\}\Big] + (1-\chi)\,\mathbb{V}^R\Big[\nu \mid \{p_i, s_i\}\Big]$$

where $\mathbb{V}^C\Big[\nu \mid \{p_i, s_i\}\Big] = (\tau_0 + \tau_s)^{-1}$ and $\mathbb{V}^R\Big[\nu \mid \{p_i, s_i\}\Big] = (\tau_0 + N\tau_s)^{-1}$. Plugging expectation and variance in the best-response function and matching coefficients, we obtain the following system of equations

$$\beta = \frac{\beta\left(\beta\rho\left(\tau_0 + \tau_s + \chi\left(N-1\right)\tau_s\right)\right) + \tau_s\left(N\left(\tau_s + \tau_0\right) + \tau_s\left(N-3\right)\tau_s\chi - 2\chi\tau_0\right)}{N\left[\left(\beta\rho\left(\tau_0 + \tau_s + \chi\left(N-1\right)\tau_s\right)\right) + 2\left(1-\chi\right)\tau_s\left(\tau_0 + \tau_s\right)\right]}$$

$$\gamma = \frac{\beta(N-2)(\tau_0 + \tau_s)(\tau_0 + N\tau_s)}{(N-1)\left((1-\chi)N\tau_s(\tau_0+\tau_s) + \beta\rho(\tau_0 + \tau_s + \chi(N-1)\tau_s)\right)}$$

One solution is $\beta = \gamma = 0$ which clearly violates the SOC. The other solution is

$$\beta = \frac{\tau_s\left(\chi(N-1)(2\tau_0 + N\tau_s) - N(\tau_0+\tau_s)\right)}{(N-1)\rho(\tau_0 + \tau_s + \chi(N-1)\tau_s)} \tag{B.1}$$

Note that

$$\beta|_{\chi=0} = -\frac{N}{N-1}\frac{\tau_s}{\rho} < 0, \beta|_{\chi=1} = \frac{N-2}{N-1}\frac{\tau_s}{\rho} > 0$$

and that

$$\frac{d}{d\chi}\beta = \frac{2\tau_s(\tau_0+\tau_s)(\tau_0+N\tau_s)}{\rho(\tau_0+\tau_s+\chi(N-1)\tau_s)^2} > 0.$$

with rational traders ($\chi = 0$, where we know no equilibrium exists) the candidate loading is negative, while the fully cursed economy always gives a positive candidate. Since the candidate equilibrium is increasing, there is a threshold above which it is positive. To check whether the candidate β actually constitute an equilibrium we need to check the SOC, which reads

$$\frac{2}{(N-1)\gamma} + \rho\left[\chi\left(\tau_0+\tau_s\right)^{-1} + (1-\chi)\left(\tau_0 + N\tau_s\right)^{-1}\right] > 0.$$

Substituting the expressions for the equilibrium γ, we obtain[13]

$$\chi(N-1)(2\tau_0 + N\tau_s) - N(\tau_0+\tau_s) > 0$$
$$\Leftrightarrow \quad \chi > \bar{\chi} \coloneqq \frac{N(\tau_0+\tau_s)}{(N-1)(2\tau_0+N\tau_s)}.$$

hence, cursedness restores equilibrium existence even without noise, provided that it is large enough. Notice that

$$\frac{\mathrm{d}}{\mathrm{d}N}\bar{\chi} = -\frac{(\tau_0+\tau_s)(2\tau_0+N^2\tau_s)}{(N-1)^2(2\tau_0+N\tau_s)^2} < 0$$

and $\lim_{N\to\infty}\bar{\chi} = 0$; the individual amount of cursedness required to sustain an equilibrium is decreasing in the number of traders, and any amount of cursedness suffices if we consider arbitrary number of traders.

[13]Imposing that the candidate β (B.1) is positive delivers the same condition $\chi > \bar{\chi}$; a positive candidate β is necessary and sufficient for existence.